Device for Conversion of Kinetic and Thermal Energies of Gas Flows to Work

Operational Principles, Comprehensive Technical Analysis, and Performance and Design Optimisation

US Patent 12,650,086 B1

2026

This document presents a comprehensive engineering analysis of the two-stage gas-flow energy-conversion device described in US Patent 12,650,086 B1 "Device for conversion of kinetic and thermal energies of gas flows to work". The device combines two turbines with sensor-controlled interstage liquid injection and thermodynamic conditioning to recover useful work from the kinetic, pressure and thermal energy of a moving gas stream. The document's sections explain the device's architecture and operating sequence; establish a comprehensive inventory of operating, thermodynamic, fluid, mechanical, geometric and control variables; and develop a mathematical model covering turbine performance, conventional spray evaporation, pressure-dependent saturation, flash evaporation, high-temperature gas-driven evaporation, coupled mass, species, momentum and energy balances, pressure losses, net power, exergy, system sizing and operational constraints. The model distinguishes between a gas state that is thermodynamically capable of supporting boiling and the occurrence of a particular nucleate- or film-boiling heat-transfer regime. It also applies property-domain, minimum-temperature, evaporation, carryover, pressure, Mach-number and numerical-convergence constraints. The document then addresses multiobjective and robust optimisation, control-system design, Programmable Logic Controller (PLC) implementation, instrumentation, equipment selection, commissioning and verification requirements.

Annexure A contains the underlying patent material, including the principal drawing, operating description and claims on which the analysis is based. Annexure B provides the detailed System Performance and Design Optimisation Simulator specification, translating the document's thermodynamic, sizing and constrained-control models into a reproducible computational framework for evaluating fixed designs, optimising operating conditions and testing performance across nominal, off-design and uncertain scenarios. The specification includes a regime-aware spray-zone model for conventional evaporation, pressure-dependent boiling conditions, isenthalpic flash evaporation and high-temperature gas-driven evaporation, together with bounded coupled iteration, property-domain checks and structured feasibility and failure statuses. The accompanying System Performance and Design Optimisation Simulator is a self-contained browser application that implements a screening-level, lumped one-dimensional version of this framework. It allows users to enter scenarios in SI or US customary units, optimise a common system geometry and scenario-specific operating parameters, compare results, review performance profiles and export calculations, logs and figures. For each operating result, it reports the local saturation temperature and latent heat, gas and liquid superheat, flash fraction and flash mass flow, gas-driven evaporation, effective evaporation coefficient, representative droplet temperature, active evaporation regime, limiting constraint and calculation status. It is intended for engineering screening, comparative analysis and design development.

Operational principles of the device for conversion of kinetic and thermal energies of gas flows to work (US Patent 12,650,086 B1)

The patent describes a two-stage gas-flow energy-conversion device that combines cascaded turbines with feedback-controlled evaporative cooling. Its intended purpose is to extract useful work from both the kinetic energy and part of the thermal energy carried by a moving gas stream. The principal arrangement is shown in the drawing on page 3, with the detailed operating description and claims on pages 4–6 of Annexure A.[1]

1. System architecture

The device contains:

- A duct with a gas-flow inlet and an outlet to the atmosphere.
- A first turbine positioned downstream of the inlet.
- A second turbine positioned downstream of the first turbine and near the outlet.
- A first measurement zone upstream of the first turbine, designated Zone 1.
- A second measurement zone between the two turbines, designated Zone 2.
- A liquid injector positioned at or immediately downstream of the first turbine.
- Pressure and temperature sensors in Zone 1, Zone 2 and the external atmosphere.
- A controller that regulates the liquid-injection rate.
- A liquid reservoir, pump, filter and associated piping.
- Mechanical or electrical loads coupled to one or both turbines.

The total useful output is intended to be the combined shaft work produced by the first and second turbines.

2. Primary gas-flow conversion

Gas enters the duct through the inlet and passes through the first turbine.

The first turbine converts part of the gas stream's kinetic energy into rotational mechanical work. That work may be used directly by machinery or converted into electricity through a generator.

As the gas passes through the first turbine:

- Its velocity and total available energy are reduced.
- A pressure difference develops across the turbine.
- The gas may also undergo a temperature reduction, depending on the expansion process and turbine configuration.

In conventional turbine systems, a considerable quantity of residual kinetic and thermal energy may remain in the downstream gas. The patent attempts to recover a greater fraction of that residual energy.[2]

3. Post-turbine evaporative cooling

The central operating feature is the injection of an evaporating liquid into the gas stream immediately after the first turbine.

The injector produces a mist of liquid droplets. As the droplets evaporate, they absorb latent heat from the gas stream. The patent states that this rapid evaporative cooling creates a reduction in gas temperature and a corresponding pressure reduction in or near Zone 2.[3]

The intended effect is to increase the effective pressure and temperature differential across the first turbine:

(1) $$\Delta P_1 = P_{\text{Zone 1}} - P_{\text{Zone 2}}$$

(2) $$\Delta T_1 = T_{\text{Zone 1}} - T_{\text{Zone 2}}$$

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[1] de St. Amatus, B. and de St. Amatus, A. (2026) *Device for conversion of kinetic and thermal energies of gas flows to work*. US Patent 12,650,086 B1, 9 June.
[2] Dixon, S.L. and Hall, C.A. (2014) *Fluid Mechanics and Thermodynamics of Turbomachinery*. 7th edn. Oxford: Butterworth-Heinemann.
[3] Sirignano, W.A. (2010) *Fluid Dynamics and Transport of Droplets and Sprays*. 2nd edn. Cambridge: Cambridge University Press.

By lowering the downstream thermodynamic state, the system is intended to increase the expansion potential across the first turbine, thereby improving its work extraction.

In engineering terms, this is a form of controlled interstage evaporative cooling. The thermal energy absorbed during evaporation is not directly converted into shaft work. Rather, the cooling is intended to alter the pressure-temperature conditions so that the turbines can extract more work from the overall flow.

4. Feedback-control principle

The injection rate is not fixed. It is regulated using real-time pressure and temperature measurements.

The controller receives data representing:

(3) $P_1, T_1, P_2, T_2, P_a, T_a$

where:

- P_1, T_1 are the pressure and temperature in Zone 1.
- P_2, T_2 are the pressure and temperature in Zone 2.
- P_a, T_a are ambient atmospheric pressure and temperature.

The controller then adjusts liquid mass flow approximately according to a control relationship of the form:

(4) $\dot{m}_l = f\left(P_1, T_1, P_2, T_2, P_a, T_a\right)$

The patent describes the intended operating relationship as:

(5) $P_1 > P_2 > P_a$

and similarly indicates that the Zone 1 temperature should remain above the Zone 2 temperature, while Zone 2 remains above the relevant ambient condition.

The controller therefore has two competing objectives:

1. Produce sufficient cooling and pressure reduction downstream of the first turbine to enhance its output.
2. Prevent the Zone 2 pressure from falling to or below atmospheric pressure, which could cause ambient air to enter through the outlet.

This closed-loop regulation is one of the most important operational aspects of the invention.[4]

5. Function of the second turbine

The second turbine has two stated functions.

Residual energy recovery

Gas leaving the evaporation zone still contains kinetic energy. The second turbine converts a further portion of that residual kinetic energy into mechanical or electrical work.

Thus:

(6) $W_{\text{gross}} = W_{\text{turbine 1}} + W_{\text{turbine 2}}$

Resistance to atmospheric backflow

Because the duct outlet is open to the atmosphere, there is a risk that a low pressure in Zone 2 could draw external air backwards into the duct.

The second turbine is positioned near the outlet and is configured to oppose or restrict this ambient-air ingress. It therefore acts both as an energy-conversion stage and as an aerodynamic flow-control element.

Strictly speaking, the turbine is not a positive shut-off valve. Its effectiveness in preventing reverse flow would depend on its blade geometry, rotational speed, loading, outlet pressure and the overall duct pressure distribution.

[4] Åström, K.J. and Murray, R.M. (2008) *Feedback Systems: An Introduction for Scientists and Engineers*. Princeton, NJ: Princeton University Press.

6. Complete operational sequence

A typical operating cycle can be described as follows:

1. A source supplies a moving gas stream to the duct inlet.
2. Zone 1 sensors measure the incoming gas pressure and temperature.
3. The gas passes through the first turbine and produces shaft work.
4. The gas enters Zone 2.
5. The injector atomises liquid into the post-turbine gas flow.
6. The liquid evaporates, absorbing heat and cooling the gas.
7. The controller compares Zone 1, Zone 2 and ambient conditions.
8. The injection rate is increased or reduced to maintain the desired pressure and temperature relationships.
9. The cooled gas passes through the second turbine.
10. The second turbine extracts residual kinetic energy and helps resist atmospheric backflow.
11. The gas exits to the atmosphere.
12. The outputs of the two turbines are combined mechanically, electrically or through separate loads.

7. Energy-conversion interpretation

The device should not be understood as creating energy from cooling. It is intended to convert energy already present in the incoming gas stream.

The available sources are:

- Bulk kinetic energy of the moving gas.
- Pressure energy or flow exergy.
- Thermal energy or enthalpy of the gas.
- Potentially, energy associated with an upstream industrial, exhaust, renewable or waste-heat process.

The evaporating liquid changes the thermodynamic state of the gas and may enable a greater expansion ratio or more favourable turbine operating condition.

A simplified net-power balance would be:

(7) $$W_{\text{net}} = W_1 + W_2 - W_{\text{pump}} - W_{\text{controller}} - W_{\text{auxiliaries}} - \text{additional flow losses}$$

Accordingly, the engineering value of the arrangement depends on whether the additional turbine output exceeds:

- Pumping power for the injected liquid.
- Injector and atomisation losses.
- Added duct pressure losses.
- Generator and transmission losses.
- Control-system consumption.
- Maintenance and liquid-supply requirements.

8. Thermodynamic qualification

The patent states that evaporation produces rapid cooling and a pressure drop. Cooling can contribute to a lower static pressure under suitable flow and geometric conditions, but the actual result is not determined by temperature alone.

Liquid injection also:

- Adds mass to the gas stream.
- Changes gas composition and molecular weight.
- Alters density.
- Changes local momentum.

- May create two-phase flow before complete evaporation.
- Introduces pressure losses through mixing and turbulence.

Therefore, the resulting Zone 2 pressure must be established through compressible-flow analysis and testing. Depending on duct area, gas velocity, evaporation rate, outlet back pressure and turbine loading, mass addition can partly offset or modify the pressure reduction expected from cooling.[5]

The operating principle is consequently credible as a controlled thermodynamic-flow concept, but the size of the net efficiency improvement cannot be determined from the patent alone.

9. Practical operating requirements

For the system to function as described, several conditions must be satisfied.

Suitable gas source

The incoming gas must possess sufficient velocity, pressure or thermal energy. A weak ambient airflow may not provide enough available energy to overcome the losses of two turbines and the injection system.

Effective evaporation

The injected liquid must evaporate within the available residence time. This requires appropriate:

- Droplet size.
- Liquid volatility.
- Gas temperature.
- Relative humidity or vapour concentration.
- Injector location.
- Duct length.
- Gas velocity.

Controlled liquid quantity

Excessive injection could cause incomplete evaporation, liquid carryover, blade erosion, fouling, corrosion or excessive pressure loss.

Stable turbine operation

Both turbines must operate within acceptable ranges for:

- Tip speed.
- Pressure ratio.
- Mass flow.
- Blade incidence.
- Rotational speed.
- Generator load.

Changes in injection rate can change gas density and mass flow, meaning the controller may also need to coordinate turbine or generator loading.

Environmental and materials compatibility

The liquid and its vapour must be compatible with:

- Turbine materials.
- Bearings and seals.
- Downstream emissions requirements.
- Occupational safety constraints.
- Local environmental regulations.

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[5] White, F.M. (2016) *Fluid Mechanics*. 8th edn. New York: McGraw-Hill Education.

10. Principal inventive concept

The invention is not simply the use of two turbines. Its operational core is the integration of:

1. A first turbine extracting energy from an incoming gas stream.
1. Feedback-controlled evaporation immediately downstream of that turbine.
2. Pressure and temperature sensing in two internal zones and in the atmosphere.
3. A control algorithm that maintains defined inter-zone and ambient relationships.
4. A second turbine that recovers residual kinetic energy while discouraging reverse atmospheric flow.

The distinctive concept is therefore best described as a sensor-controlled, two-stage gas turbine system with interstage evaporative thermodynamic conditioning.

Overall assessment

Operationally, the device seeks to create a managed pressure-temperature cascade:

$$\text{Zone 1} \rightarrow \text{First turbine} \rightarrow \text{Evaporative cooling in Zone 2} \rightarrow \text{Second turbine} \rightarrow \text{Atmosphere} \quad (8)$$

The first turbine performs the primary conversion, the evaporative system modifies the downstream state to improve the available differential, and the second turbine recovers remaining flow energy while stabilising the atmospheric outlet boundary.

The patent provides a coherent high-level control and energy-conversion architecture. However, it does not provide enough quantitative information, such as gas mass flow, pressure ratios, liquid type, evaporation rate, turbine efficiency or parasitic power, to establish the net output, efficiency gain or commercially optimal operating range. Those matters require a further thermodynamic and fluid-dynamics analysis.

Variable inventory under constant atmospheric pressure and temperature

Assume the atmospheric boundary conditions are fixed:

(9) $$P_a = \text{constant}, T_a = \text{constant}$$

The patent expressly varies the internal pressures and temperatures, liquid-injection rate and turbine operating conditions. A complete engineering model, however, requires additional flow, geometry, fluid-property, control and loss variables. The inventory below covers a practical lumped-parameter, one-dimensional or CFD model of the patented device.[6]

1. Time and operating state

Symbol	Variable
t	Time
x	Axial position in the duct
r, θ	Radial and circumferential position for CFD
i	Operating zone or station index
$\mathrm{d}(\ )/\mathrm{d}t$	Rate of change of any transient state
X_0	Initial condition of each state variable

Every pressure, temperature, flow rate, shaft speed and injection parameter may be time-dependent.

2. Incoming gas variables

At the duct inlet or Zone 1:

Symbol	Variable
$P_{g,in}$ or P_1	Gas static pressure
$T_{g,in}$ or P_{1e}, T_{1e}, P_2, T_2	Gas static temperature
$P_{0,in}$	Gas stagnation or total pressure
$T_{0,in}$	Gas stagnation or total temperature
$\dot{m}_g$	Gas mass-flow rate
Q_g	Gas volumetric-flow rate
u_{in}	Mean gas velocity
u_x, u_r, u_θ	Axial, radial and swirl velocity components
ρ_1	Gas density
v_1	Gas specific volume
h_1	Specific enthalpy
$u_{int,1}$	Specific internal energy
s_1	Specific entropy
$e_{k,1}$	Specific kinetic energy
M_1	Mach number
Re_1	Reynolds number
Pr_1	Prandtl number
μ_1	Dynamic viscosity
ν_1	Kinematic viscosity

[6] de St. Amatus, B. and de St. Amatus, A. (2026) *Device for conversion of kinetic and thermal energies of gas flows to work*. US Patent 12,650,086 B1, 9 June.

Symbol	Variable
k_1	Thermal conductivity
$c_{p,1}$	Specific heat at constant pressure
$c_{v,1}$	Specific heat at constant volume
γ_1	Specific-heat ratio
R_g	Gas-specific gas constant
$Y_{j,1}$	Mass fraction of each gas constituent
$x_{j,1}$	Mole fraction of each gas constituent
ϕ_1	Relative humidity or vapour saturation ratio
ω_1	Humidity ratio
I_1	Turbulence intensity
$k_{turb,1}$	Turbulent kinetic energy
ε_1	Turbulent dissipation rate
$\ell_{turb,1}$	Turbulence length scale
α_1	Gas-flow angle into the first turbine

The gas-source variables may also include pulsation frequency, pressure fluctuation amplitude and temperature fluctuation amplitude.[7]

3. First-turbine inlet and outlet variables

Thermodynamic variables

Symbol	Variable
$P_{1,in}$	First-turbine inlet pressure
$P_{1,out}$	First-turbine outlet pressure
$T_{1,in}$	First-turbine inlet temperature
$T_{1,out}$	First-turbine outlet temperature
ΔP_{T1}	Pressure drop across turbine 1
ΔT_{T1}	Temperature drop across turbine 1
π_{T1}	Turbine 1 pressure ratio
Δh_{T1}	Actual specific enthalpy drop
$\Delta h_{T1,s}$	Isentropic specific enthalpy drop
$\dot{m}_{T1}$	Gas mass flow through turbine 1
$u_{T1,in}$	Inlet gas velocity
$u_{T1,out}$	Outlet gas velocity
$M_{T1,in}$	Inlet Mach number
$M_{T1,out}$	Outlet Mach number

[7] Çengel, Y.A., Boles, M.A. and Kanoğlu, M. (2019) *Thermodynamics: An Engineering Approach*. 9th edn. New York: McGraw-Hill Education.

Mechanical variables

Symbol	Variable
N_1	Rotational speed in revolutions per minute
Ω_1	Angular speed of turbine 1
τ_1	Shaft torque
$\dot{W}_{T1}$	Turbine 1 shaft power
$\eta_{is,1}$	Isentropic efficiency
$\eta_{m,1}$	Mechanical efficiency
$\eta_{v,1}$	Volumetric or flow efficiency
$\eta_{g,1}$	Generator efficiency
λ_1	Tip-speed ratio
$U_{tip,1}$	Blade-tip speed
ψ_1	Stage-loading coefficient
φ_1	Flow coefficient
$C_{Q,1}$	Torque coefficient
$C_{P,1}$	Power coefficient
J_1	Rotor moment of inertia
$\dot{\Omega}_1$	Rotor angular acceleration of turbine 1
L_1	Applied mechanical or electrical load
$V_{gen,1}$	Generator voltage
$I_{gen,1}$	Generator current
$f_{gen,1}$	Generator electrical frequency
$T_{bearing,1}$	Bearing temperature
$F_{thrust,1}$	Axial thrust force
$F_{radial,1}$	Radial bearing load
$A_{vib,1}$	Vibration amplitude

4. Zone 2 variables

Zone 2 is the region between the turbines in which liquid is injected and evaporated.

Symbol	Variable
P_2	Zone 2 gas pressure
T_2	Zone 2 gas temperature
$P_{0,2}$	Zone 2 total pressure
$T_{0,2}$	Zone 2 total temperature
ρ_2	Gas-mixture density
u_2	Mean axial gas velocity
$u_{\theta,2}$	Residual swirl velocity
$\dot{m}_{g,2}$	Dry-gas mass-flow rate
$\dot{m}_{v,2}$	Vapour mass-flow rate
$\dot{m}_{mix,2}$	Total gas-vapour mass-flow rate
Q_2	Volumetric-flow rate

Symbol	Variable
h_2	Mixture specific enthalpy
s_2	Mixture specific entropy
M_2	Mach number
Re_2	Reynolds number
ϕ_2	Relative humidity or saturation ratio
ω_2	Vapour or humidity ratio
$Y_{v,2}$	Vapour mass fraction
$Y_{l,2}$	Suspended-liquid mass fraction
α_g	Gas volume fraction
α_l	Liquid volume fraction
I_2	Turbulence intensity
$k_{turb,2}$	Turbulent kinetic energy
ε_2	Turbulence dissipation rate
$t_{res,2}$	Gas residence time in Zone 2
ΔP_{mix}	Pressure loss due to injection and mixing
ΔP_{evap}	Pressure change associated with cooling and evaporation
ΔT_{evap}	Evaporative temperature reduction
q_{evap}	Heat-transfer rate into the evaporating liquid
$\dot{Q}_{wall,2}$	Heat transfer through the Zone 2 duct wall

The principal controlled relationships described by the patent are:

(10) $\Delta P_{12} = P_1 - P_2$

(11) $\Delta T_{12} = T_1 - T_2$

(12) $\Delta P_{2a} = P_2 - P_a$

(13) $\Delta T_{2a} = T_2 - T_a$

Because P_a and T_a are fixed, the four differentials vary through P_1, T_1, P_2, T_2 .

5. Injected-liquid variables

Liquid supply variables

Symbol	Variable
L	Liquid identity or formulation
T_l	Liquid supply temperature
P_l	Liquid supply pressure
ρ_l	Liquid density
μ_l	Liquid viscosity
ν_l	Liquid kinematic viscosity
$c_{p,l}$	Liquid specific heat
k_l	Liquid thermal conductivity
σ_l	Surface tension

Symbol	Variable
h_{fg}	Latent heat of vaporisation
$P_{vap}(T)$	Vapour pressure
T_b	Normal boiling temperature
$P_{crit,\ell}$	Liquid critical pressure
$T_{crit,\ell}$	Liquid critical temperature
$T_{sat,2} = T_{sat}(P_2)$	Saturation temperature of the injected liquid at the Zone 2 static pressure
$h_{fg,2} = h_{fg}(P_2)$	Latent heat of vaporisation at the Zone 2 pressure
$h_{\ell,in}$	Specific enthalpy of the supplied liquid
$\Delta T_{sup,\ell} = T_{\ell,in} - T_{sat,2}$	Liquid superheat relative to the Zone 2 saturation temperature
$M_{prop,\ell}$	Selected liquid-property method or formulation
$D_{prop,\ell}$	Valid pressure–temperature domain of the selected liquid-property method
M_l	Molecular mass
$D_{v\text{-}g}$	Vapour diffusivity in the gas
V_{res}	Reservoir liquid volume
m_{res}	Reservoir liquid mass
H_{res}	Reservoir liquid level

Injection variables

Symbol	Variable
$\dot{m}_l$	Commanded liquid mass-flow rate
Q_l	Liquid volumetric-flow rate
$P_{pump,out}$	Pump discharge pressure
$P_{nozzle,in}$	Pressure immediately upstream of the injector orifice
ΔP_{nozzle}	$P_{nozzle,in} - P_2$ pressure differential across the injector orifice
ΔP_{inj}	Pressure differential across injector
u_l	Droplet injection velocity
C_d	Injector discharge coefficient
$A_{orifice}$	Effective injector-orifice area
N_{inj}	Number of injector nozzles
θ_{spray}	Spray-cone angle
x_{inj}	Injector axial location
r_{inj}	Injector radial location
α_{inj}	Injection orientation
f_{inj}	Pulsed-injection frequency
D_{32}	Sauter mean droplet diameter
D_{10}	Arithmetic mean droplet diameter
D_{v90}	Upper droplet-size percentile
$p(D)$	Complete droplet-size distribution

Symbol	Variable
N_d	Droplet number density
We	Weber number
Oh	Ohnesorge number
Re_d	Droplet Reynolds number
η_{atom}	Atomisation efficiency
SMD	Spray mean diameter, equivalent to D_{32}

Evaporation variables

Symbol	Variable
$\dot{m}_{evap}$	Liquid evaporation rate
χ_{evap}	Fraction of injected liquid evaporated
$m_{l,unevap}$	Unevaporated liquid mass
$\dot{m}_{carry}$	Liquid carryover rate
t_{evap}	Droplet evaporation time
T_d	Droplet temperature
T_{wb}	Local wet-bulb temperature
Sh	Sherwood number
Nu_d	Droplet Nusselt number
B_M	Spalding mass-transfer number
B_T	Spalding heat-transfer number
K_{evap}	Droplet evaporation constant
$A_{drop,total}$	Total droplet surface area
$\dot{Q}_{latent}$	Latent heat absorption rate
$\dot{Q}_{sensible,l}$	Sensible heat absorbed by the liquid
$S_{supersat}$	Vapour supersaturation ratio
$\dot{m}_{cond}$	Condensation rate, if local condensation occurs
$\Delta T_{sup,g} = T_{1e} - T_{sat,2}$	Gas superheat at entry to Zone 2
X_{flash}	Initial isenthalpic flash fraction
$\dot{m}_{flash}$	Net flash-vapour mass-flow rate accepted by the final state
$\dot{m}_{flash,ret}$	Excess flash vapour returned to the liquid inventory
$\dot{m}_{evap,g}$	Gas-driven evaporation mass-flow rate after flash evaluation
K_{eff}	Effective high-temperature evaporation coefficient
$\dot{C}_g = \dot{m}_{da}\left(c_{pa} + \omega_1 c_{pv}\right)$	Dry-air-basis gas heat-capacity rate
R_{evap}	Active evaporation regime
L_{evap}	Active evaporation limiting constraint
S_{iter}	Coupled energy–mass iteration status

A principal control input is:

(14) $\dot{m}_l = \text{variable}$

The effective thermodynamic input is the portion of the injected liquid that actually evaporates:

(15) $$\dot{m}_{evap} = \chi_{evap}\dot{m}_l$$

6. Pump, reservoir, filter and pipe variables

Symbol	Variable
N_p	Pump rotational speed
ΔP_p	Pump pressure rise
$\dot{W}_p$	Pump power
η_p	Pump efficiency
H_p	Pump head
$NPSH_a$	Available net positive suction head
$NPSH_r$	Required net positive suction head
$C_{v,valve}$	Valve flow coefficient
x_v	Control-valve position
ΔP_{pipe}	Pipe pressure loss
ΔP_{filter}	Filter pressure loss
R_f	Filter flow resistance
f_{pipe}	Pipe friction factor
D_{pipe}	Pipe internal diameter
L_{pipe}	Pipe length
ε_{pipe}	Pipe roughness
T_{pipe}	Pipe or liquid-line temperature
C_{line}	Hydraulic compliance
τ_{line}	Fluid-delivery time constant
$t_{delay,l}$	Delay between command and spray delivery
C_{contam}	Liquid contamination concentration

7. Second-turbine variables

The second turbine variables are equivalent to those of turbine 1 but carry subscript 2.

Symbol	Variable
$P_{2,in}$	Turbine 2 inlet pressure
$P_{2,out}$	Turbine 2 outlet pressure
$T_{2,in}$	Turbine 2 inlet temperature
$T_{2,out}$	Turbine 2 outlet temperature
ΔP_{T2}	Pressure drop across turbine 2
ΔT_{T2}	Temperature drop across turbine 2
π_{T2}	Turbine 2 pressure ratio
$\dot{m}_{T2}$	Mixture mass-flow rate through turbine 2
N_2	Rotational speed
Ω_2	Angular speed of turbine 2
τ_2	Shaft torque

Symbol	Variable
$\dot{W}_{T2}$	Turbine 2 shaft power
$\eta_{is,2}$	Isentropic efficiency
$\eta_{m,2}$	Mechanical efficiency
$\eta_{g,2}$	Generator efficiency
$U_{tip,2}$	Blade-tip speed
λ_2	Tip-speed ratio
ψ_2	Stage-loading coefficient
φ_2	Flow coefficient
J_2	Rotor moment of inertia
L_2	Applied generator or mechanical load
$F_{thrust,2}$	Axial thrust
$T_{bearing,2}$	Bearing temperature
$A_{vib,2}$	Vibration amplitude
$K_{reverse,2}$	Effective resistance to atmospheric reverse flow
$\dot{m}_{reverse}$	Reverse-flow mass rate
$f_{reverse}$	Reverse-flow fraction

8. Outlet variables

Although atmospheric pressure and temperature are fixed, the gas immediately inside the outlet can vary.

Symbol	Variable
P_{out}	Internal static pressure immediately before discharge
T_{out}	Gas discharge temperature
$P_{0,out}$	Discharge total pressure
$T_{0,out}$	Discharge total temperature
u_{out}	Outlet velocity
$\dot{m}_{out}$	Outlet mass-flow rate
ρ_{out}	Outlet-mixture density
M_{out}	Outlet Mach number
$Y_{v,out}$	Vapour mass fraction
$Y_{l,out}$	Liquid-droplet mass fraction
ϕ_{out}	Outlet relative humidity
α_{out}	Discharge-flow angle
p'_{out}	Outlet-pressure fluctuation
u'_{out}	Outlet-velocity fluctuation
L_{noise}	Acoustic sound-pressure level
$\dot{m}_{emission,j}$	Emission rate of each gas or vapour species

9. Duct and system geometry variables

Symbol	Variable
L_d	Total duct length
D_h	Hydraulic diameter
$A(x)$	Local duct cross-sectional area
$D(x)$	Local duct diameter
$P_w(x)$	Wetted perimeter
ε_d	Internal surface roughness
t_w	Duct-wall thickness
k_w	Duct-wall thermal conductivity
$c_{p,w}$	Wall specific heat
ρ_w	Wall material density
$T_w(x,t)$	Duct-wall temperature
L_{Z1}	Length of Zone 1
L_{Z2}	Length of Zone 2
V_{Z1}	Zone 1 internal volume
V_{Z2}	Zone 2 internal volume
S_{12}	Spacing between turbines
$S_{1,inj}$	Distance from turbine 1 to injector
$S_{inj,2}$	Distance from injector to turbine 2
$S_{2,out}$	Distance from turbine 2 to outlet
β_d	Duct expansion or contraction angle
$K_{minor,j}$	Minor-loss coefficient of each fitting
f_D	Darcy friction factor
ΔP_{duct}	Total duct pressure loss
A_{leak}	Effective leakage area
$\dot{m}_{leak}$	Duct leakage mass-flow rate

10. Turbine design variables

For each turbine $j = 1,2$:

Symbol	Variable
$D_{rotor,j}$	Rotor diameter
$D_{hub,j}$	Hub diameter
$D_{tip,j}$	Tip diameter
$N_{b,j}$	Number of blades
$c_j(r)$	Blade chord distribution
$s_j(r)$	Blade spacing
$\beta_j(r)$	Blade-angle or twist distribution
$p_j(r)$	Blade pitch distribution
$t_{blade,j}$	Blade thickness

Symbol	Variable
$A_{blade,j}$	Blade planform area
$\sigma_{sol,j}$	Rotor solidity
$g_{tip,j}$	Blade-tip clearance
$A_{annulus,j}$	Rotor annulus area
B_j	Effective blockage ratio
$C_{L,j}$	Blade lift coefficient
$C_{D,j}$	Blade drag coefficient
$C_{loss,j}$	Aerodynamic loss coefficient
$\eta_{tip,j}$	Tip-loss factor
$\eta_{hub,j}$	Hub-loss factor
$\theta_{stator,j}$	Stator or guide-vane setting
$x_{pitch,j}$	Variable blade-pitch position, when applicable

11. Sensor variables

The patent specifically uses pressure and temperature gauges in Zone 1, Zone 2 and the atmosphere.

For each pressure sensor:

Symbol	Variable
$P_{meas,i}$	Measured pressure
$b_{P,i}$	Pressure-sensor bias
$n_{P,i}$	Pressure-measurement noise
$S_{P,i}$	Pressure-sensor sensitivity
$\tau_{P,i}$	Pressure-sensor time constant
$R_{P,i}$	Pressure-sensor range
$e_{P,i}$	Pressure-sensor calibration error
$f_{s,P}$	Pressure-sampling frequency

For each temperature sensor:

Symbol	Variable
$T_{meas,i}$	Measured temperature
$b_{T,i}$	Temperature-sensor bias
$n_{T,i}$	Temperature-measurement noise
$S_{T,i}$	Temperature-sensor sensitivity
$\tau_{T,i}$	Temperature-sensor time constant
$R_{T,i}$	Temperature-sensor range
$e_{T,i}$	Temperature-sensor calibration error
$f_{s,T}$	Temperature-sampling frequency

The measured and true values are related by expressions such as:

(16) $$P_{meas,i} = P_i + b_{P,i} + n_{P,i}$$

(17) $$T_{meas,i} = T_i + b_{T,i} + n_{T,i}$$

12. Controller variables

Symbol	Variable
$\dot{m}_{l,set}$	Liquid-injection command
$P_{2,set}$	Desired Zone 2 pressure
$T_{2,set}$	Desired Zone 2 temperature
$\Delta P_{12,set}$	Desired Zone 1–Zone 2 pressure difference
$\Delta T_{12,set}$	Desired Zone 1–Zone 2 temperature difference
P_{margin}	Minimum pressure margin above atmosphere
T_{margin}	Desired temperature margin relative to atmosphere
e_P	Pressure-control error
e_T	Temperature-control error
K_P	Proportional controller gain
K_I	Integral controller gain
K_D	Derivative controller gain
K_{ff}	Feedforward gain
u_c	Controller output signal
x_I	Controller integral state
$\dot{m}_{l,min}$	Minimum permitted injection rate
$\dot{m}_{l,max}$	Maximum permitted injection rate
$r_{\dot{m}}$	Maximum injection-rate ramp
t_s	Controller sampling interval
t_d	Computational or communication delay
f_c	Controller update frequency
P_{trip}	Low-pressure trip threshold
T_{trip}	Low-temperature or icing trip threshold
$N_{trip,1}, N_{trip,2}$	Turbine overspeed trip thresholds
S_{mode}	Controller operating mode
C_{logic}	Control-law or algorithm configuration

A general controller representation is:

(18) $$\dot{m}_{l,set} = f\left(P_1, T_1, P_2, T_2, \dot{m}_g, N_1, N_2, P_a, T_a\right)$$

with P_a and T_a fixed, the remaining arguments are variable.

13. Heat-transfer variables

Symbol	Variable
$\dot{Q}_{g\text{-}l}$	Heat transfer from gas to liquid
$\dot{Q}_{g\text{-}w}$	Heat transfer from gas to duct wall
$\dot{Q}_{w\text{-}a}$	Heat transfer from wall to atmosphere
$h_{g\text{-}l}$	Gas–droplet convective heat-transfer coefficient
$h_{g\text{-}w}$	Gas–wall heat-transfer coefficient
$h_{w\text{-}a}$	External wall–atmosphere heat-transfer coefficient

Symbol	Variable
A_{ht}	Available heat-transfer area
q''	Local heat flux
Bi	Biot number
Nu	Nusselt number
ε_{rad}	Surface emissivity
$\dot{Q}_{rad}$	Radiative heat-transfer rate
T_{wall}	Local wall temperature
T_{film}	Local film temperature

14. Mass-balance variables

Symbol	Variable
$m_{g,Z1}$	Gas mass stored in Zone 1
$m_{g,Z2}$	Gas or mixture mass stored in Zone 2
$\mathrm{d}m_{g,Z1} / \mathrm{d}t$	Zone 1 mass accumulation
$\mathrm{d}m_{g,Z2} / \mathrm{d}t$	Zone 2 mass accumulation
$\dot{m}_{in}$	Gas inflow
$\dot{m}_{out}$	Mixture outflow
$\dot{m}_{l}$	Injected liquid flow
$\dot{m}_{evap}$	Evaporated-liquid flow
$\dot{m}_{drain}$	Collected or drained liquid flow
$\dot{m}_{leak}$	Leakage flow
$\dot{m}_{reverse}$	Atmospheric reverse flow

The complete mass balance is approximately:

(19) $$\frac{\mathrm{d}m_{system}}{\mathrm{d}t} = \dot{m}_{in} + \dot{m}_{l} + \dot{m}_{reverse} - \dot{m}_{out} - \dot{m}_{drain} - \dot{m}_{leak}$$

15. Energy and power variables

Symbol	Variable
$\dot{E}_{kin,in}$	Incoming kinetic-energy rate
$\dot{E}_{pressure,in}$	Incoming pressure-flow energy rate
$\dot{H}_{in}$	Incoming enthalpy rate
$\dot{E}_{thermal,in}$	Incoming thermal-energy rate
$\dot{W}_{T1}$	First-turbine gross power
$\dot{W}_{T2}$	Second-turbine gross power
$\dot{W}_{gross}$	Combined turbine power
$\dot{W}_{p}$	Pump electrical demand
$\dot{W}_{c}$	Controller and sensor power
$\dot{W}_{aux}$	All non-pump auxiliary electrical demand
$\dot{W}_{bearing}$	Bearing-friction loss
$\dot{W}_{windage}$	Rotor windage loss

Symbol	Variable
$\dot{W}_{genloss}$	Generator losses
$\dot{W}_{net}$	Net useful power
η_{sys}	Overall energy-conversion efficiency
η_{ex}	Exergy efficiency
$\dot{X}_{dest}$	Exergy-destruction rate
$\dot{S}_{gen}$	Entropy-generation rate
E_{stored}	Energy stored in the duct and rotating machinery

The principal output variable is:

(20) $$\dot{W}_{net} = \dot{W}_{T1} + \dot{W}_{T2} - \dot{W}_{p} - \dot{W}_{c} - \dot{W}_{aux} - \dot{W}_{loss}$$

16. Structural and mechanical variables

Symbol	Variable
σ_{blade}	Blade stress
ε_{blade}	Blade strain
T_{blade}	Blade temperature
δ_{blade}	Blade deformation
f_n	Natural frequency
f_{exc}	Excitation frequency
ζ	Structural damping ratio
A_{vib}	Vibration amplitude
N_{cycles}	Operating fatigue cycles
D_f	Fatigue-damage fraction
$r_{erosion}$	Droplet-erosion rate
$r_{corrosion}$	Corrosion rate
$r_{fouling}$	Fouling rate
$g_{bearing}$	Bearing clearance
$\mu_{bearing}$	Bearing friction coefficient
T_{lub}	Lubricant temperature
P_{lub}	Lubricant pressure

17. Atmospheric variables that can still vary

Fixing atmospheric pressure and temperature does not automatically fix every atmospheric property. Unless separately held constant, the following remain variables:

Symbol	Variable
ϕ_a	Ambient relative humidity
ω_a	Ambient humidity ratio
$Y_{j,a}$	Ambient gas composition
ρ_a	Ambient density if composition or humidity varies
u_a	External wind velocity at the outlet
α_a	External wind direction

Symbol	Variable
p'_a	Local atmospheric pressure fluctuation around the specified constant mean
$C_{cont,a}$	Ambient contaminant concentration
I_a	Ambient turbulence intensity

For a strictly constant atmospheric boundary, these would also need to be fixed.

18. Performance, optimisation and constraint variables

Possible objective variables

- Maximum net power, $\dot{W}_{net}$
- Maximum gross turbine power, $\dot{W}_{gross}$
- Maximum overall efficiency, η_{sys}
- Maximum exergy efficiency, η_{ex}
- Minimum liquid consumption, $\dot{m}_l$
- Minimum pumping power, $\dot{W}_p$
- Minimum pressure loss, ΔP_{loss}
- Minimum entropy generation, $\dot{S}_{gen}$
- Minimum reverse flow, $\dot{m}_{reverse}$
- Minimum liquid carryover, $\dot{m}_{carry}$
- Minimum erosion, corrosion and fouling
- Minimum acoustic emission
- Maximum operational stability

Possible constraint variables

- $P_2 > P_a$
- $T_2 > T_a$, where required by the patent's stated operating relationship
- $N_1 < N_{1,max}$
- $N_2 < N_{2,max}$
- $\dot{m}_l \leq \dot{m}_{l,max}$
- f_{evap} above a specified minimum
- $\dot{m}_{carry}$ below an allowable maximum
- Blade stress below allowable stress
- Bearing temperature below its limit
- No icing or condensation in prohibited regions
- No compressor-like reverse operation of either turbine
- Stable outlet flow without atmospheric ingestion

Compact minimum variable set

For a first lumped-parameter simulation, the minimum useful variable vector would be:

(21) $$x = \left[P_1, T_1, P_2, T_2, \dot{m}_g, u_1, u_2, \rho_1, \rho_2, N_1, N_2, \tau_1, \tau_2, \dot{m}_l, \dot{m}_{evap}, D_{32}, T_l, P_{inj}, \dot{W}_{T1}, \dot{W}_{T2}, \dot{W}_p, \dot{W}_{net}\right)$$

with fixed:

(22) $$P_a, T_a$$

and design parameters such as duct dimensions, turbine geometry and injector geometry either fixed for one simulation or treated as optimisation variables.

The distinction is important: operating variables change while the device runs; design variables are normally fixed after construction but can be varied during design optimisation; and fluid-property variables change indirectly with pressure, temperature, composition and phase.

Comprehensive mathematical model

The patented system consists of a first turbine, an interstage liquid-injection and evaporative-cooling zone, a second turbine near the atmospheric outlet, pressure and temperature sensing, and a controller regulating the injection rate. The model below converts that architecture into a constrained dynamic optimisation problem.[8]

Atmospheric pressure and temperature are fixed:

(23) $$P_a = P_a, T_a = T_a$$

All other operating, control and design quantities may vary.

1. System stations

Define four principal stations:

(24) $$1: \text{Zone 1, upstream of turbine 1}$$

(25) $$1e: \text{outlet of turbine 1, before liquid injection}$$

(26) $$2: \text{Zone 2, after evaporation and before turbine 2}$$

(27) $$3: \text{outlet of turbine 2, upstream of the outlet duct, screen, diffuser and final atmospheric - discharge losses}$$

The required nominal pressure and temperature hierarchy is:

(28) $$P_1 > P_2 > P_a$$

(29) $$T_1 > T_2 > T_a$$

The outlet station normally satisfies:

(30) $$P_3 = P_{\mathrm{a}} + \Delta P_{\mathrm{exit}} \geq P_a$$

At the final atmospheric-discharge plane, the static pressure is P_a. The final atmospheric plane is a boundary condition and is not included as a dynamic state.

2. Variable classification

2.1 State vector

A suitable lumped dynamic state vector is

(31) $$x(t) = \begin{bmatrix} P_1 & T_1 & P_{1e} & T_{1e} & P_2 & T_2 & P_3 & T_3 \\ M_1 & M_2 & M_{v,2} & M_{l,2} & \Omega_1 & \Omega_2 & T_{w,1} & T_{w,2} \end{bmatrix}^T$$

where:

- M_k is the gas or gas–vapour mass stored in zone k;
- $M_{v,2}$ is the vapour mass in Zone 2;
- $M_{l,2}$ is the suspended unevaporated liquid mass;
- Ω_j is turbine angular velocity of turbine j;
- $T_{w,k}$ is a representative duct-wall temperature.

2.2 Control vector

(32) $$u(t) = \left[\dot{m}_l \quad P_{inj} \quad \tau_{g,1} \quad \tau_{g,2} \quad x_v \quad \beta_1 \quad \beta_2\right)^T$$

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[8] de St. Amatus, B. and de St. Amatus, A. (2026) *Device for conversion of kinetic and thermal energies of gas flows to work*. US Patent 12,650,086 B1, 9 June.

where:

- $\dot{m}_l$ is the injected-liquid mass-flow rate;
- P_{inj} is injector supply pressure;
- $\tau_{g,j}$ is generator or mechanical load torque;
- x_v is liquid-valve position;
- β_j is an optional blade-pitch or guide-vane setting.

2.3 Disturbance vector

(33) $$w(t) = \begin{bmatrix} P_{1,in} & T_{1,in} & \dot{m}_{g,in} & Y_{v,in} & u_{in} & I_{turb,in} \end{bmatrix}^T$$

These represent varying source-gas conditions.

2.4 Design vector

(34) $$d = \begin{bmatrix} A(x) & L_d & D_h & D_{T1} & D_{T2} & N_{b,1} & N_{b,2} & A_{inj} & D_{32} & V_1 & V_2 \end{bmatrix}^T$$

Design variables are fixed during a particular operating run but may be varied during optimisation.

3. Governing dynamic model

The complete model has differential-algebraic form:

(35) $$F(\dot{x}, x, u, d, w) = 0$$

subject to:

(36) $$g(x, u, d, w) \leq 0$$

(37) $$h(x, u, d, w) = 0$$

4. Thermodynamic state equations

For each gas or gas–vapour zone:

(38) $$P_k = \rho_k R_{mix,k} T_k$$

(39) $$\rho_k = \frac{M_k}{v_k}$$

(40) $$R_{mix,k} = \sum_j Y_{j,k} R_j$$

(41) $$h_k = h(P_k, T_k, Y_k)$$

(42) $$u_k = h_k - \frac{P_k}{\rho_k}$$

(43) $$s_k = s(P_k, T_k, Y_k)$$

For a calorically perfect approximation:

(44) $$h_k = c_{p,k} T_k$$

(45) $$u_k = c_{v,k} T_k$$

(46) $$\gamma_k = \frac{c_{p,k}}{c_{v,k}}$$

The gas velocity at a station follows from continuity:

(47) $$U_k = \frac{\dot{m}_k}{\rho_k A_k}$$

and the Mach number is:

(48) $$Ma_k = \frac{U_k}{\sqrt{\gamma_k R_{mix,k} T_k}}$$

5. Zone mass balances

For Zone 1:

(49) $$\frac{\mathrm{d}M_1}{\mathrm{d}t} = \dot{m}_{in} - \dot{m}_{T1}$$

For Zone 2:

(50) $$\frac{\mathrm{d}M_2}{\mathrm{d}t} = \dot{m}_{T1} + \dot{m}_{evap} - \dot{m}_{T2} - \dot{m}_{leak} + \dot{m}_{reverse}$$

Suspended liquid balance:

(51) $$\frac{\mathrm{d}M_{l,2}}{\mathrm{d}t} = \dot{m}_l - \dot{m}_{evap} - \dot{m}_{carry} - \dot{m}_{drain}$$

Vapour balance:

(52) $$\frac{\mathrm{d}M_{v,2}}{\mathrm{d}t} = \dot{m}_{v,in} + \dot{m}_{evap} - Y_{v,2}\dot{m}_{T2}$$

At steady state:

(53) $$\dot{m}_{T2} = \dot{m}_{T1} + \dot{m}_{evap} - \dot{m}_{leak} + \dot{m}_{reverse}$$

and:

(54) $$\dot{m}_l = \dot{m}_{evap} + \dot{m}_{carry} + \dot{m}_{drain}$$

6. Zone energy balances

6.1 Zone 1

(55) $$\frac{\mathrm{d}}{\mathrm{d}t}\left(M_1 u_1\right) = \dot{m}_{in}\left(h_{in} + \frac{V_{in}^2}{2}\right) - \dot{m}_{T1}\left(h_1 + \frac{V_1^2}{2}\right) + \dot{Q}_{w,1}$$

6.2 Evaporative Zone 2

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[M_2 u_2 + M_{l,2} u_l\right) = \dot{m}_{T1}\left(h_{1e} + \frac{U_{1e}^2}{2}\right)$$

$$+\dot{m}_l\left(h_{l,in} + \frac{U_l^2}{2}\right)$$

$$-\dot{m}_{T2}\left(h_2 + \frac{U_2^2}{2}\right)$$

(56)

$$-\dot{m}_{carry}\left(h_{l,2} + \frac{U_{l,2}^2}{2}\right)$$

$$+\dot{Q}_{w,2}$$

At steady state:

(57) $$\dot{m}_{T1}\left(h_{1e} + \frac{U_{1e}^2}{2}\right) + \dot{m}_l h_{l,in} + \dot{Q}_{w,2} = \dot{m}_{T2}\left(h_2 + \frac{U_2^2}{2}\right)$$

$$+\dot{m}_{carry} h_{l,2}$$

If the carryover and drained-liquid streams leave at the same temperature and negligible velocity, their enthalpy terms may be combined. They must not be omitted from the control-volume energy balance.

The latent heat absorbed by evaporation is:

(58) $$\dot{Q}_{latent} = \dot{m}_{evap} h_{fg}(T_2)$$

and the sensible heat absorbed by the liquid is:

(59) $$\dot{Q}_{sens,l} = \dot{m}_l c_{p,l}\left(T_2 - T_{l,in}\right)$$

Therefore, approximately:

(60) $$\dot{m}_{T1} c_{p,g}\left(T_{1e} - T_2\right) = \dot{m}_{evap} h_{fg} + \dot{m}_l c_{p,l}\left(T_2 - T_{l,in}\right) + \dot{Q}_{loss}$$

This equation directly links the controlled liquid-injection rate to T_2.[9]

7. First-turbine model

For turbine 1, define the isentropic outlet state by:

(61) $$s_{1e,s} = s_1$$

(62) $$h_{1e,s} = h\left(P_{1e}, s_1\right)$$

The actual outlet enthalpy is:

(63) $$h_{1e} = h_1 - \eta_{is,1}\left(h_1 - h_{1e,s}\right)$$

[9] Incropera, F.P., DeWitt, D.P., Bergman, T.L. and Lavine, A.S. (2007) *Fundamentals of Heat and Mass Transfer*. 6th edn. Hoboken, NJ: John Wiley & Sons.

The turbine power is:

(64) $$\dot{W}_{T1} = \eta_{m,1}\dot{m}_{T1}\left[h_1 - h_{1e} + \frac{V_1^2 - V_{1e}^2}{2}\right)$$

The turbine torque is:

(65) $$\tau_{T1} = \frac{\dot{W}_{T1}}{\Omega_1}$$

For an ideal-gas approximation:

(66) $$T_{1e,s} = T_1\left(\frac{P_{1e}}{P_1}\right)^{(\gamma_1\text{-}1)/\gamma_1}$$

(67) $$T_{1e} = T_1 - \eta_{is,1}\left(T_1 - T_{1e,s}\right)$$

(68) $$\dot{W}_{T1} = \eta_{m,1}\dot{m}_{T1}c_{p,1}\left(T_1 - T_{1e}\right)$$

The first-turbine pressure ratio is:

(69) $$\Pi_1 = \frac{P_1}{P_{1e}}$$

8. Evaporation model

8.1 Evaporated fraction

Define:

(70) $$\dot{m}_{evap} = \chi_{evap}\dot{m}_l$$

where:

(71) $$0 \le \chi_{evap} \le 1$$

Using a D^2 -law droplet model:

(72) $$\frac{\mathrm{d}D^2}{\mathrm{d}t} = -K_{evap}$$

For residence time t_{res} :

(73) $$D_f^2 = \max\left(D_0^2 - K_{evap}t_{res}, 0\right)$$

Because droplet mass is proportional to D^3 :

(74) $$\chi_{evap} = 1 - \left(\frac{D_f}{D_0}\right)^3$$

or:

(75) $$\chi_{evap} = 1 - \left[\max\left(1 - \frac{K_{evap}t_{res}}{D_0^2}, 0\right)\right)^{3/2}$$

The residence time is approximately:

(76) $$t_{res} = \frac{L_2}{V_2}$$

The evaporation coefficient may be represented by:

$$K_{evap} = K_{evap}\left(T_2, P_2, Y_{v,2}, D_0, Re_d, Sh, Nu_d\right) \quad (77)$$

8.2 Saturation limitation

For an air–water approximation:

$$\omega_{sat}\left(T_2, P_2\right) = 0.622 \frac{P_{sat}\left(T_2\right)}{P_2 - P_{sat}\left(T_2\right)} \quad (78)$$

The maximum vapour addition is:

$$\dot{m}_{evap,\max} = \dot{m}_{dry} \max\left[\omega_{sat}\left(T_2, P_2\right) - \omega_{in}, 0\right) \quad (79)$$

Therefore:

$$\dot{m}_{evap} = \min\left[\chi_{evap}\dot{m}_l, \dot{m}_{evap,\max}\right] \quad (80)$$

The liquid carryover is:

$$\dot{m}_{carry} = \max\left(\dot{m}_l - \dot{m}_{evap} - \dot{m}_{drain}, 0\right) \quad (81a)$$

8.3 Boiling, flash evaporation and high-temperature treatment

The D^2 -law and saturation-capacity treatment in Sections 8.1–8.2 applies to conventional droplet evaporation. A separate regime-aware treatment is required whenever the gas entering Zone 2 or the supplied liquid can exceed the pressure-dependent saturation temperature at the Zone 2 static pressure. The normal boiling temperature is not sufficient because the local boiling temperature varies with pressure.

The local saturation temperature is defined by:

$$P_{sat}\left(T_{sat,2}\right) = P_2, P_2 < P_c \quad (81b)$$

The gas-side and liquid-side superheats are:

$$\Delta T_{g,sh} = \max\left(T_{1e} - T_{sat,2}, 0\right), \Delta T_{\ell,sh} = \max\left(T_{\ell,in} - T_{sat,2}, 0\right) \quad (81c)$$

The latent heat shall be evaluated at the local saturation state. A bounded Watson-type relation is suitable for screening when a validated liquid-property routine is not available:

$$h_{fg,2} = h_{fg,r}\left[\frac{1 - T_{sat,2}/T_c}{1 - T_r/T_c}\right)^{n_W} \quad (81d)$$

Flash evaporation is determined from an isenthalpic pressure reduction from the liquid-supply state to P_2 . For a constant-liquid-heat-capacity approximation:

$$x_{flash} = \text{clip}\left[\frac{h_\ell\left(T_{\ell,in}\right) - h_f\left(P_2\right)}{h_{fg,2}}, 0, 1\right) \approx \text{clip}\left[\frac{c_{p\ell}\left(T_{\ell,in} - T_{sat,2}\right)}{h_{fg,2}}, 0, 1\right) \quad (81e)$$

$$\dot{m}_{flash} = x_{flash}\dot{m}_\ell, \dot{m}_{\ell,r} = \left(1 - x_{flash}\right)\dot{m}_\ell, T_{\ell,r} = \min\left(T_{\ell,in}, T_{sat,2}\right) \quad (81f)$$

The conventional D^2 kinetic limit remains active. In a high-temperature gas it may be accelerated by a bounded multiplier rather than by allowing an unbounded or discontinuous evaporation coefficient:

$$K_{eff} = K_{evap}F_{HT}, F_{HT} = \min\left(F_{HT,max}, 1 + \frac{\Delta T_{g,sh}}{\Delta T_{HT,ref}}\right) \quad (81g)$$

$$\chi_{kin} = 1 \quad \left[\max\left(1 - \frac{K_{eff}t_{res}}{D_0^2}, 0\right)\right)^{3/2} \quad (81h)$$

The remaining liquid is heated sensibly towards a representative droplet temperature bounded by the local gas and saturation temperatures:

(81i) $$T_d = \max\left[T_{\ell,r}, \min\left(T_2, T_{sat,2}\right)\right], \dot{Q}_{sens} = \dot{m}_{\ell,r} c_{p\ell}\left(T_d - T_{\ell,r}\right)$$

The gas-driven evaporation is also limited by the sensible energy that may be removed while preserving the minimum permitted Zone 2 temperature. Define the dry-air-basis heat-capacity rate as $\dot{C}_g = \dot{m}_{da}\left(c_{pa} + \omega_1 c_{pv}\right)$. Then:

(81j) $$\dot{m}_{evap,E} = \max\left[\frac{\dot{C}_g\left(T_{1e} - T_{2,min}\right) - \dot{Q}_{sens}}{h_{fg,2}}, 0\right)$$

The saturation-capacity constraint is finite only when the final gas state is below the local boiling boundary. An arbitrary clamp of saturation pressure to a fixed fraction of total pressure shall not be used:

(81k) $$\dot{m}_{evap,sat} = \begin{cases} \dot{m}_{da}\max\left[\omega_{sat}\left(T_2, P_2\right) - \omega_1, 0\right), & P_{sat}\left(T_2\right) < P_2, \\ +\infty, & P_{sat}\left(T_2\right) \geq P_2. \end{cases}$$

The gas-driven and total net evaporation rates are:

(81l) $$\dot{m}_{evap,g} = min\left(\chi_{kin}\dot{m}_{\ell,r}, \dot{m}_{evap,E}, \dot{m}_{evap,sat}\right), \dot{m}_{evap} = \dot{m}_{flash} + \dot{m}_{evap,g} \leq \dot{m}_{\ell}$$

Flash vapour obtains its latent energy from the entering liquid enthalpy. Only the gas-driven evaporation and liquid sensible heating are therefore charged to the Zone 2 gas-side energy balance:

(81m) $$\dot{C}_g\left(T_{1e} - T_2\right) = \dot{Q}_{sens} + \dot{m}_{evap,g} h_{fg,2} + \dot{Q}_{loss,2}$$

For reporting and control, the active evaporation regime is classified as:

(81n) $$R = \begin{cases} \text{conventional}, & T_{1e} < T_{sat,2} \text{ and } T_{\ell,in} \leq T_{sat,2}, \\ \text{high - temperature or boiling - capable}, & T_{1e} \geq T_{sat,2} \text{ and } T_{\ell,in} \leq T_{sat,2}, \\ \text{flash - assisted}, & T_{\ell,in} > T_{sat,2}. \end{cases}$$

A boiling-capable gas state does not by itself prove that nucleate or film boiling occurs. The accepted evaporation rate remains limited by residence time, energy availability, saturation or superheated-state logic, carryover and the specified minimum Zone 2 temperature. The solver shall reject states at or above the liquid critical pressure, outside the selected property correlation range, or with a failed coupled energy–mass iteration. It shall return $T_{sat,2}$, $h_{fg,2}$, flash fraction, effective evaporation coefficient, active regime and the limiting constraint with every result.

9. Pressure and momentum model in Zone 2

The pressure reduction cannot be determined from temperature alone. It follows from the coupled continuity, energy and momentum equations.

For a constant-area evaporative control volume:

(82) $$P_{1e}A + \dot{m}_{T1}V_{1e} + F_{spray,x} = P_2 A + \dot{m}_{T2}V_2 + F_f$$

where:

(83) $$F_f = K_{mix}\frac{\rho_2 V_2^2}{2}A + f_D\frac{L_2}{D_h}\frac{\rho_2 V_2^2}{2}A$$

Hence:

(84) $$P_2 = P_{1e} + \frac{\dot{m}_{T1}V_{1e} - m_{T2}V_2}{A} + \frac{F_{spray,x} - F_f}{A}$$

The density change is:

(85) $$\rho_2 = \frac{P_2}{R_{mix,2} T_2}$$

and:

(86) $$V_2 = \frac{\dot{m}_{T2}}{\rho_2 A}$$

Thus, P_2, T_2, ρ_2, V_2, $\dot{m}_{evap}$ and $\dot{m}_l$ must be solved simultaneously.

10. Second-turbine model

The second-turbine equations are:

(87) $$s_{3,s} = s_2$$

(88) $$h_{3,s} = h(P_3, s_2)$$

(89) $$h_3 = h_2 - \eta_{is,2}(h_2 - h_{3,s})$$

(90) $$\dot{W}_{T2} = \eta_{m,2}\dot{m}_{T2}\left[h_2 - h_3 + \frac{V_2^2 - V_3^2}{2}\right)$$

(91) $$\tau_{T2} = \frac{\dot{W}_{T2}}{\Omega_2}$$

The pressure ratio is:

(92) $$\Pi_2 = \frac{P_2}{P_3}$$

For an ideal-gas approximation:

(93) $$T_{3,s} = T_2\left(\frac{P_3}{P_2}\right)^{(\gamma_2\text{-}1)/\gamma_2}$$

(94) $$T_3 = T_2 \text{ - } \eta_{is,2}(T_2 - T_{3,s})$$

11. Turbine mass-flow maps

To avoid independently prescribing both pressure ratio and mass flow, each turbine should use a performance map:

(95) $$\dot{m}_{Tj,corr} = \Phi_j(N_{j,corr}, \Pi_j, \beta_j)$$

where:

(96) $$\dot{m}_{Tj,corr} = \dot{m}_{Tj}\frac{\sqrt{T_{0,in,j}/T_{ref}}}{P_{0,in,j}/P_{ref}}$$

and:

(97) $$N_{j,corr} = \frac{N_j}{\sqrt{T_{0,in,j}/T_{ref}}}$$

Therefore:

(98) $$\dot{m}_{Tj} = \frac{P_{0,in,j}}{P_{ref}} \sqrt{\frac{T_{ref}}{T_{0,in,j}}} \Phi_j \left(N_{j,corr}, \Pi_j, \beta_j \right)$$

Map-domain constraints are:

(99) $$\dot{m}_{j,min} \le \dot{m}_{Tj} \le \dot{m}_{j,max}$$

(100) $$\Pi_{j,min} \le \Pi_j \le \Pi_{j,max}$$

(101) $$N_{j,corr,min} \le N_{j,corr} \le N_{j,corr,max}$$

12. Rotor and generator dynamics

For turbine j:

(102) $$J_j \frac{\mathrm{d}\Omega_j}{\mathrm{d}t} = \tau_{Tj} - \tau_{g,j} - \tau_{fr,j}$$

The electrical output is:

(103) $$\dot{W}_{el,j} = \eta_{g,j} \tau_{g,j} \Omega_j$$

Bearing and windage losses may be represented by:

(104) $$\tau_{fr,j} = c_{0,j} + c_{1,j} \omega_j + c_{2,j} \Omega_j^2$$

The gross electrical output is:

(105) $$\dot{W}_{gross} = \dot{W}_{el,1} + \dot{W}_{el,2}$$

13. Pump and injection-system model

The pump pressure rise must satisfy:

(106a) $$\Delta P_{nozzle,in} \ge P_2 + \Delta P_{nozzle}$$

(106b) $$\Delta P_{pump,out} \ge P_{nozzle,in} + \Delta P_{line} + \Delta P_{filter}$$

(106c) $$\Delta P_p \ge P_{pump,out} - P_{reservoir} + \Delta P_{suction}$$

where $\Delta P_{suction}$ is a positive suction-side loss.

The liquid volumetric flow is:

(107) $$Q_l = \frac{\dot{m}_l}{\rho_l}$$

Pump power is:

(108) $$\dot{W}_p = \frac{\dot{m}_l \Delta P_p}{\rho_l \eta_p}$$

Injector flow may be represented by:

(109) $$\dot{m}_l = C_d A_{orifice} \sqrt{2 \rho_l \left(P_{nozzle,in} - P_2 \right)}$$

subject to:

(110) $$P_{nozzle,in} > P_2$$

14. Duct-loss model

For duct section r:

(111) $$\Delta P_{f,r} = f_r \frac{L_r}{D_{h,r}} \frac{\rho_r V_r^2}{2}$$

Minor losses are:

(112) $$\Delta P_{m,r} = K_r \frac{\rho_r V_r^2}{2}$$

Total parasitic pressure loss is:

(113) $$\Delta P_{par,gas} = \sum_r \left(\Delta P_{f,r} + \Delta P_{m,r}\right) + \Delta P_{mix} + \Delta P_{spray,gas}$$

This excludes the intended pressure drops across the turbines. $\Delta P_{spray,gas}$ is the gas-side pressure loss caused by spray penetration, droplet acceleration and phase mixing. It is distinct from the liquid-side nozzle pressure differential ΔP_{nozzle}.

15. Reverse-flow model

A simple atmospheric reverse-flow model is:

(114) $$\dot{m}_{reverse} = C_{rev} A_{rev} \sqrt{2\rho_a \max\left(P_a - P_3, 0\right)}$$

An equivalent smooth approximation is:

(115) $$\dot{m}_{reverse} = C_{rev} A_{rev} \sqrt{2\rho_a \mathrm{softplus}\left(P_a - P_3\right)}$$

A strict no-reverse-flow condition is:

(116) $$P_3 \geq P_a + \Delta P_{3a,min}$$

or:

(117) $$\dot{m}_{reverse} = 0$$

The aerodynamic resistance of turbine 2 can be incorporated through:

(118) $$A_{rev} = A_{out} \Psi_{rev}\left(\Omega_2, \beta_2, \text{blade geometry}\right)$$

with:

(119) $$0 \leq \Psi_{rev} \leq 1$$

16. Net-power objective

The principal output objective is:

(120) $$\dot{W}_{net} = \dot{W}_{el,1} + \dot{W}_{el,2} - \dot{W}_p - \dot{W}_{aux}$$

Expanded:

(121) $$\dot{W}_{net} = \eta_{g,1}\eta_{m,1}\dot{m}_{T1}\left(h_1 - h_{1e} + \frac{U_1^2 - U_{1e}^2}{2}\right)$$

$$+\eta_{g,2}\eta_{m,2}\dot{m}_{T2}\left(h_2 - h_3 + \frac{U_2^2 - U_3^2}{2}\right)$$

$$-\frac{\dot{m}_l \Delta P_p}{\rho_l \eta_p \eta_{motor,p}} - \dot{W}_{aux}$$

This directly links net output to:

- P_1, T_1, P_{1e}, T_{1e} ;
- P_2, T_2, P_3, T_3 ;
- turbine pressure ratios and efficiencies;
- injected-liquid flow;
- evaporation rate;
- pump pressure and efficiency;
- turbine speeds and electrical loads.

17. Boundary-dependent first-law performance ratio

Define the energy rate entering with the source gas as:

(122) $$\dot{E}_{in} = \dot{m}_{in}\left[h_1 - h_a + \frac{V_1^2 - V_a^2}{2}\right)$$

If liquid enters at a state different from ambient:

(123) $$\dot{E}_{l,in} = \dot{m}_l\left[h_l\left(T_{l,in}\right) - h_l\left(T_a\right)\right)$$

Define the following boundary-dependent first-law performance ratio:

(124) $$\Pi_I = \frac{\dot{W}_{net}}{\dot{E}_{in} + \dot{E}_{l,in}}$$

This ratio shall not be assumed to lie between zero and one:

(125) $$0 \leq \Pi_I \leq 1$$

unless the selected control-volume boundary includes all external pressure, thermal, kinetic, liquid and auxiliary-energy inputs on a consistent reference-state basis. The exergy efficiency in Section 18 is the preferred overall thermodynamic performance metric.

18. Exergy efficiency

With fixed P_a, T_a , the specific physical flow exergy is:

(126) $$e_x = \left(h - h_a\right) - T_a\left(s - s_a\right) + \frac{V^2}{2}$$

The incoming gas exergy rate is:

(127) $$\dot{X}_{in} = \dot{m}_{in}\left[(h_1 - h_a) - T_a(s_1 - s_a) + \frac{V_1^2}{2}\right)$$

Liquid physical exergy is:

(128) $$\dot{X}_{l,in} = \dot{m}_l\left[(h_l - h_{l,a}) - T_a(s_l - s_{l,a})\right)$$

The exergy efficiency is:

(129) $$\eta_{ex} = \frac{\dot{W}_{net}}{\dot{X}_{in} + \dot{X}_{l,in}}$$

The exergy balance is:

(130) $$\dot{X}_{in} + \dot{X}_{l,in} = \dot{W}_{net} + \dot{X}_{out} + \dot{X}_{dest}$$

where:

(131) $$\dot{X}_{dest} = T_a\dot{S}_{gen}$$

19. Entropy generation objective

For the complete steady-state system:

(132) $$\dot{S}_{gen} = \sum_{out}\dot{m}_o s_o - \sum_{in}\dot{m}_i s_i$$

$$-\sum_b \frac{\dot{Q}_b}{T_b}$$

Expanded approximately:

(133) $$\dot{S}_{gen} = \dot{m}_{out}s_3 + \dot{m}_{carry}s_{l,carry}$$

$$-\dot{m}_{in}s_1 - \dot{m}_l s_{l,in}$$

$$-\frac{\dot{Q}_{wall}}{T_{wall}}$$

The second law requires:

(134) $$\dot{S}_{gen} \geq 0$$

Minimising $\dot{S}_{gen}$ also minimises exergy destruction:

(135) $$\min\dot{S}_{gen} \Leftrightarrow min\dot{X}_{dest}$$

because T_a is constant.[10]

20. Liquid-consumption objective

Total liquid consumption over horizon $0 \leq t \leq t_f$ is:

(136) $$M_l = \int_0^{t_f} \dot{m}_l(t)\mathrm{d}t$$

[10] Bejan, A. (2016) *Advanced Engineering Thermodynamics*. 4th edn. Hoboken, NJ: John Wiley & Sons.

The specific liquid consumption is:

(137) $$SLC = \frac{\dot{m}_l}{\dot{W}_{net}}$$

or, over a horizon:

(138) $$SLC_E = \frac{\int_0^{t_f} \dot{m}_l \mathrm{d}t}{\int_0^{t_f} \dot{W}_{net} \mathrm{d}t}$$

The optimisation objective is:

(139) $$\min M_l$$

or:

(140) $$\min SLC_E$$

subject to the required cooling and pressure-differential constraints.

21. Carryover, erosion and durability objectives

21.1 Carryover fraction

(141) $$f_{carry} = \frac{\dot{m}_{carry}}{\dot{m}_l}$$

Constraint:

(142) $$f_{carry} \le f_{carry,max}$$

21.2 Droplet-erosion rate

A general empirical model is:

(143) $$\dot{E}_{erosion,j} = K_{e,j}\dot{m}_{carry} D_{32}^a V_{rel,j}^b \left[\sin\left(\alpha_{imp,j}\right)\right)^c$$

where:

(144) $$V_{rel,j} = \left|V_{droplet} - V_{blade,j}\right)$$

The durability constraint is:

(145) $$\dot{E}_{erosion,j} \le \dot{E}_{erosion,j,max}$$

or over the design life:

(146) $$\int_0^{t_{life}} \dot{E}_{erosion,j} \mathrm{d}t \le E_{allow,j}$$

22. Structural constraints

A simplified centrifugal blade stress model is:

(147) $$\sigma_{cent,j} = C_{\sigma,j}\rho_{blade,j}\Omega_j^2\left(r_{tip,j}^2 - r_{hub,j}^2\right)$$

Total equivalent stress may be represented by:

(148) $$\sigma_{eq,j} = \sqrt{\sigma_{cent,j}^2 + 3\tau_{aero,j}^2}$$

Constraint:

(149) $$\sigma_{eq,j} \le \frac{\sigma_{yield,j}}{SF_j}$$

Tip-speed constraint:

(150) $$U_{tip,j} = \Omega_j r_{tip,j} \le U_{tip,j,max}$$

Overspeed constraint:

(151) $$\Omega_j \le \Omega_{j,max}$$

23. Thermal and phase constraints

The patent's preferred thermodynamic hierarchy gives:

(152) $$P_1 - P_2 \ge \Delta P_{12,min}$$

(153) $$P_2 - P_a \ge \Delta P_{2a,min}$$

(154) $$T_1 - T_2 \ge \Delta T_{12,min}$$

(155) $$T_2 - T_a \ge \Delta T_{2a,min}$$

Additional thermal constraints are:

(156) $$T_2 \ge T_{freeze} + \Delta T_{freeze}$$

(157) $$T_{blade,j} \le T_{blade,j,max}$$

(158) $$T_{bearing,j} \le T_{bearing,j,max}$$

To prevent supersaturation where condensation is unacceptable:

(159) $$Y_{v,2} \le Y_{v,sat}(P_2, T_2)$$

or:

(160) $$\phi_2 \le 1$$

To require substantially complete evaporation:

(161) $$\chi_{evap} \ge \chi_{evap,min}$$

24. Injection and pump constraints

(162) $$0 \le \dot{m}_l \le \dot{m}_{l,max}$$

(163) $$\left|\frac{\mathrm{d}\dot{m}_l}{\mathrm{d}t}\right) \le r_{\dot{m},max}$$

(164) $$P_{inj,min} \le P_{inj} \le P_{inj,max}$$

(165) $$P_{inj} - P_2 \ge \Delta P_{inj,min}$$

(166) $$NPSH_a \ge NPSH_r + \Delta NPSH_{margin}$$

(167) $$\dot{W}_p \le \dot{W}_{p,max}$$

25. Turbine-operating constraints

For $j = 1,2$:

(168) $$\Omega_{j,min} \le \Omega_j \le \Omega_{j,max}$$

(169) $$\tau_{j,min} \le \tau_j \le \tau_{j,max}$$

(170) $$\dot{W}_{j,min} \le \dot{W}_{Tj} \le \dot{W}_{j,max}$$

(171) $$\Pi_{j,min} \le \Pi_j \le \Pi_{j,max}$$

(172) $$Ma_{tip,j} = \frac{\Omega_j r_{tip,j}}{\sqrt{\gamma_j R_j T_j}} \le Ma_{tip,max}$$

If the turbine map identifies stall or unstable regions:

(173) $$SM_j \ge SM_{j,min}$$

where SM_j is an operating-margin measure.

26. Outlet and reverse-flow constraints

(174) $$P_3 - P_a \ge \Delta P_{out,min}$$

(175) $$\dot{m}_{out} \ge 0$$

(176) $$\dot{m}_{reverse} \le \dot{m}_{reverse,max}$$

(177) $$\frac{\dot{m}_{reverse}}{\dot{m}_{out}} \le f_{reverse,max}$$

For strict prevention:

(178) $$\dot{m}_{reverse} = 0$$

27. Acoustic objective and constraint

A reduced-order acoustic model may be written as:

(179) $$L_p = L_{ref} + 10\log_{10}\left[C_1\left(\frac{U_{tip,1}}{U_{ref}}\right)^{n_1} + C_2\left(\frac{U_{tip,2}}{U_{ref}}\right)^{n_2} + C_3\left(\frac{\Delta P_{inj}}{P_{ref}}\right)^{n_3}\right)$$

Constraint:

(180) $$L_p \le L_{p,max}$$

Objective:

(181) $$\min L_p$$

28. Dynamic-stability constraints

Linearising the closed-loop model around an operating point gives:

(182) $$\delta\dot{\mathrm{x}} = A_{cl}\delta x + B_{cl}\delta \mathrm{w}$$

Asymptotic stability requires:

(183) $$\max_i \Re\left[\lambda_i\left(A_{cl}\right)\right) < 0$$

A specified stability margin is:

(184a) $\max_{i} \Re\left[\lambda_i\left(A_{cl}\right)\right] \leq -\alpha_{min}$

For the discrete PLC implementation, let the linearised sampled system be:

(184b) $\delta x_{k+1} = A_d \delta x_k + B_{cl} \delta w_k$

Discrete-time asymptotic stability requires:

(184c) $\max_{i} \left|\lambda_i\left(A_d\right)\right| < 1$

The existing negative-real-part condition applies only to the continuous-time model.

Additional transient constraints may include:

(185) $\left|P_2 - P_{2,set}\right) \leq e_{P,max}$

(186) $\left|T_2 - T_{2,set}\right) \leq e_{T,max}$

(187) $\left|\frac{\mathrm{d}P_2}{\mathrm{d}t}\right) \leq r_{P,max}$

(188) $\left|\frac{\mathrm{d}T_2}{\mathrm{d}t}\right) \leq r_{T,max}$

(189) $\left|\frac{\mathrm{d}\Omega_j}{\mathrm{d}t}\right) \leq \alpha_{\omega,j,max}$

29. Controller model

The pressure and temperature deviations below may be monitored independently. They shall not be summed with identical control signs because increased liquid injection affects the corresponding pressure and temperature differentials in different directions. Zone 2 pressure shall be the primary fast-loop controlled variable:

(190) $e_P = \left(P_1 - P_2\right) - \Delta P_{12,set}$

(191) $e_{Pa} = \left(P_2 - P_a\right) - \Delta P_{2a,set}$

(192) $e_T = \left(T_1 - T_2\right) - \Delta T_{12,set}$

(193) $e_{Ta} = \left(T_2 - T_a\right) - \Delta T_{2a,set}$

The injection command may be:

(194a) $e_2 = P_2 - P_{2,set}$

(194b) $u_{valve,raw} = u_{ff} + K_P e_2 + K_I \int e_2 \mathrm{d}t + K_D \frac{\mathrm{d}e_2}{\mathrm{d}t} + b_l$

For a normalised command:

(195a) $u_{valve,cmd} = \mathrm{clip}\left(u_{valve,raw}, 0, 1\right)$

(195b) $\dot{m}_l = f_{valve}\left(u_{valve,cmd}, N_p, P_{noozle,in}, P_2\right)$

Temperature, saturation, carryover and first-stage differential requirements shall be implemented as supervisory constraints, feedforward corrections or separately signed control terms whose process gains have been experimentally identified.

A more appropriate optimisation-based controller is:

$$\min_{u_{0:N\ 1}} \sum_{k=0}^{N\ 1} \ell(x_k, u_k) + V_f(x_N) \tag{196}$$

subject to the complete model and constraints over the prediction horizon.[11]

30. Multiobjective optimisation formulation

The unscaled objective vector is:

$$J = \begin{bmatrix} -\dot{W}_{net} \\ -\eta_I \\ -\eta_{ex} \\ \dot{m}_l \\ \dot{W}_p \\ \Delta P_{par} \\ \dot{S}_{gen} \\ \dot{m}_{reverse} \\ \dot{m}_{carry} \\ \dot{E}_{erosion} \\ L_p \\ I_{unstable} \end{bmatrix} \tag{197}$$

where negative signs convert maximisation objectives into minimisation objectives.

A dimensionless scalarised objective is:

$$\begin{aligned} J_{total} &= -w_P \frac{\dot{W}_{net}}{\dot{W}_{ref}} - w_I \frac{\eta_I}{\eta_{I,ref}} - w_X \frac{\eta_{ex}}{\eta_{ex,ref}} \\ &+ w_l \frac{\dot{m}_l}{\dot{m}_{l,ref}} + w_p \frac{\dot{W}_p}{\dot{W}_{p,ref}} + w_{\Delta P} \frac{\Delta P_{par}}{\Delta P_{ref}} \\ &+ w_S \frac{\dot{S}_{gen}}{\dot{S}_{ref}} + w_R \frac{\dot{m}_{reverse}}{\dot{m}_{ref}} + w_C \frac{\dot{m}_{carry}}{\dot{m}_{l,ref}} \\ &+ w_E \frac{\dot{E}_{erosion}}{\dot{E}_{ref}} + w_N \frac{L_p}{L_{p,ref}} + w_U I_{unstable} \end{aligned} \tag{198}$$

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[11] Rawlings, J.B., Mayne, D.Q. and Diehl, M. (2017) *Model Predictive Control: Theory, Computation, and Design*. 2nd edn. Madison, WI: Nob Hill Publishing.

with:

(199) $w_i \geq 0$

The optimisation is:

(200) $\min_{x,u,d} J_{total}$

subject to:

(201) $F(\dot{x}, x, u, d, w) = 0$

(202) $g(x, u, d, w) \leq 0$

(203) $h(x, u, d, w) = 0$

31. Preferred ε-constraint formulation

Because weighted objectives can conceal trade-offs, the most defensible engineering formulation is usually:

(204) $\max \dot{W}_{net}$

subject to:

(205) $\eta_{ex} \geq \eta_{ex,min}$

(206) $\dot{m}_l \leq \dot{m}_{l,max}$

(207) $\dot{W}_p \leq \dot{W}_{p,max}$

(208) $\dot{S}_{gen} \leq \dot{S}_{gen,max}$

(209) $\dot{m}_{carry} \leq \dot{m}_{carry,max}$

(210) $\dot{m}_{reverse} \leq \dot{m}_{reverse,max}$

(211) $\dot{E}_{erosion} \leq \dot{E}_{erosion,max}$

(212) $L_p \leq L_{p,max}$

plus all pressure, temperature, turbine, structural, phase and stability constraints.

32. Direct linkage between objectives and constraints

Objective	Direct constraint dependence
Maximise $\dot{W}_{net}$	$P_1 - P_{1e}$, $P_2 - P_3$, turbine speed, torque, efficiency, pump-power and injection limits
Maximise η_I	Available inlet energy, parasitic losses, pump power, carryover and outlet kinetic-energy loss
Maximise η_{ex}	Entropy generation, mixing loss, evaporation irreversibility and exhaust exergy
Minimise $\dot{m}_l$	Required ΔT_{12} , required ΔP_{12} , saturation limit and minimum evaporation fraction
Minimise $\dot{W}_p$	Injection pressure, injector area, pipe losses and required liquid flow
Minimise ΔP_{par}	Duct geometry, friction, spray momentum loss, mixing and blockage
Minimise $\dot{S}_{gen}$	Turbine inefficiency, finite-temperature heat transfer, mixing, friction and evaporation
Minimise reverse flow	$P_3 - P_a$, turbine-2 speed, loading and reverse-flow resistance
Minimise carryover	Droplet diameter, residence time, evaporation coefficient and saturation capacity
Minimise erosion	Carryover, droplet diameter, impact speed, blade speed and impingement angle
Minimise noise	Rotor tip speed, pressure ratio, blade loading and injector pressure
Maximise stability	Controller gains, actuator limits, sensor delay, rotor inertia and operating-map margins

33. Robust optimisation under variable inlet conditions

Because all non-atmospheric operating parameters may vary, let the disturbance set be:

(213) $w \in W$

where:

(214) $$W = \begin{Bmatrix} P_{1,min} \le P_1 \le P_{1,max} \\ T_{1,min} \le T_1 \le T_{1,max} \\ \dot{m}_{min} \le \dot{m}_{in} \le \dot{m}_{max} \\ Y_{v,min} \le Y_{v,in} \le Y_{v,max} \end{Bmatrix}$$

A robust design is:

(215) $$\max_{u,d} \min_{w \in W} \dot{W}_{net}$$

subject to:

(216) $$g(x,u,d,w) \le 0 \forall w \in W$$

This requires the pressure, temperature, phase, turbine, structural and reverse-flow constraints to remain satisfied for every specified inlet condition.[12]

34. Steady-state optimisation model

For an initial engineering study, set:

(217) $\dot{x} = 0$

The nonlinear-programming problem becomes:

(218) $$\begin{aligned} \max_{x,u,d} \quad & \dot{W}_{net} \\ \text{subject to} \quad & \text{mass balances} \\ & \text{momentum balances} \\ & \text{evaporation equations} \\ & P_1 - P_2 \ge \Delta P_{12,min} \\ & T_1 - T_2 \ge \Delta T_{12,min} \\ & P_3 - P_a \ge \Delta P_{out,min} \\ & \chi_{evap} \ge \chi_{evap,min} \\ & \dot{m}_{reverse} \le \dot{m}_{reverse,max} \\ & \sigma_{eq,j} \le \sigma_{allow,j} \end{aligned}$$

This formulation mathematically links the system's desired output, principally net power, efficiency and exergy recovery, to the pressure, temperature, liquid-use, phase, structural, operational and atmospheric-ingress constraints.

[12] Ben-Tal, A., El Ghaoui, L. and Nemirovski, A. (2009) *Robust Optimization*. Princeton, NJ: Princeton University Press.

35. Calculation scope, required inputs and scale closure

The following equations extend the steady-state model into a design-and-performance calculation that determines electrical output and the principal system dimensions from specified inlet and ambient conditions. The calculation treats the incoming flow as humid air unless a different gas composition is supplied. Humid-air properties should be evaluated consistently on either a dry-air or a humid-mixture basis; the equations below use a dry-air basis for psychrometric balances and a total-mixture basis for momentum and turbine calculations.[13]

Define the measured input vector as:

(219) $$z_{in} = \left[U_1 \quad \phi_1 \quad P_1 \quad T_1 \quad P_a \quad T_a \quad \phi_a \quad F_P \quad t_{op}\right)^T$$

where U_1 is incoming mean velocity, ϕ_1 is incoming relative humidity, P_1 and T_1 are incoming absolute pressure and temperature, P_a , T_a and ϕ_a are ambient absolute pressure, temperature and relative humidity, F_P is the prescribed Zone 2 pressure factor, and t_{op} is the operating duration.

The prescribed Zone 2 pressure is:

(220) $$P_{2,set} = F_P P_a$$

with:

(221) $$F_P > 1$$

and the control requirement is:

(222a) $$P_{2,\mathrm{set}} = F_P P_a$$

(222b) $$P_{2,\mathrm{trip}} = F_{trip} P_a \quad 1 < F_{trip} < F_P$$

(222c) $$\left|P_2(t) - P_{2,set}\right| \leq \Delta P_{control,allow}$$

during normal steady operation, and:

(222d) $$P_2(t) > P_{2,trip}$$

as the hard protective boundary.

$P_{2,\mathrm{set}}$ is the normal regulation target. $P_{2,\mathrm{trip}}$ is the lower protective threshold. A normal transient deviation below the setpoint is not itself a trip unless the measured pressure reaches the trip threshold or violates another commissioned protective limit.

The inlet state variables alone do not establish a unique physical scale. If geometrically similar machines operate at the same velocities and pressure ratios, mass flow and power scale approximately with area, whereas characteristic diameters scale with the square root of area. Therefore, one additional scale-setting input is required:

(223) $$S \in \left\{A_1, D_1, \dot{m}_{da}, \dot{W}_{net,target}, E_{el,target}\right)$$

The model can consequently be solved in either of two modes:

1. Forward-performance mode: inlet area or diameter is specified and electrical power is calculated.
2. Inverse-sizing mode: required net electrical power or energy is specified and the necessary mass flow, area and dimensions are calculated.

The steady target power corresponding to a required electrical energy is:

(224) $$\dot{W}_{net,target} = \frac{E_{el,target}}{t_{op}}$$

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[13] ASHRAE (2025) *2025 ASHRAE Handbook—Fundamentals*. Atlanta, GA: American Society of Heating, Refrigerating and Air-Conditioning Engineers, Chapter 1, ‘Psychrometrics’.

36. Psychrometric preprocessing of the incoming and ambient states

Let temperatures in thermodynamic equations be expressed in Kelvin and define the corresponding Celsius temperatures as:

(225) $$\theta_1 = T_1 - 273.15, \theta_a = T_a - 273.15$$

Let $p_{ws}(T)$ denote the saturation vapour pressure of water at temperature T. For engineering implementation, p_{ws} and other humid-air properties should be obtained from an ASHRAE-compliant correlation or a validated real-fluid property routine such as the NIST humid-air formulation.[14]

The incoming and ambient water-vapour partial pressures are:

(226) $$p_{v,1} = \phi_1 p_{ws}(T_1)$$

(227) $$p_{v,a} = \phi_a p_{ws}(T_a)$$

The corresponding humidity ratios, in kilograms of water vapour per kilogram of dry air, are:

(228) $$\omega_1 = \varepsilon \frac{p_{v,1}}{P_1 - p_{v,1}}$$

(229) $$\omega_a = \varepsilon \frac{p_{v,a}}{P_a - p_{v,a}}$$

where:

(230) $$\varepsilon = \frac{M_v}{M_{da}} \approx 0.621945$$

The dry-air and vapour densities at the inlet are:

(231) $$\rho_{da,1} = \frac{P_1 - p_{v,1}}{R_{da} T_1}$$

(232) $$\rho_{v,1} = \frac{p_{v,1}}{R_v T_1}$$

and the total humid-air density is:

(233) $$\rho_1 = \rho_{da,1} + \rho_{v,1}$$

The ambient humid-air density is similarly:

(234) $$\rho_a = \frac{P_a - p_{v,a}}{R_{da} T_a} + \frac{p_{v,a}}{R_v T_a}$$

For a perfect-mixture approximation, the inlet moist-air specific heat on a kilogram-of-dry-air basis is:

(235) $$c_{p,ma,1} = c_{p,da} + \omega_1 c_{p,v}$$

and the inlet moist-air enthalpy on the same basis is:

(236) $$h_{ma,1} = c_{p,da}\theta_1 + \omega_1 \left(h_{fg,0} + c_{p,v}\theta_1\right)$$

For turbine calculations on a total-mixture basis, the effective gas constant and specific heat are:

(237) $$R_{m,1} = \frac{R_{da} + \omega_1 R_v}{1 + \omega_1}$$

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[14] Bell, I.H., Lemmon, E.W. and Harvey, A.H. (2017) 'Algorithms for the calculation of psychrometric properties from multi-fluid Helmholtz-energy-explicit models', *International Journal of Refrigeration*, 84, pp. 59–70.

(238) $$c_{p,m,1} = \frac{c_{p,da} + \omega_1 c_{p,v}}{1+\omega_1}$$

and:

(239) $$\gamma_1 = \frac{c_{p,m,1}}{c_{p,m,1} - R_{m,1}}$$

37. Pressure allocation and pressure-factor feasibility

Define the internal outlet pressure immediately downstream of the second turbine as:

(240) $$P_3 = P_a + \Delta P_{exit}$$

where $\Delta P_{exit} \geq 0$ represents the pressure needed to overcome the outlet duct, screen, diffuser and discharge losses.

The available pressure drop across the second turbine is:

(241) $$\Delta P_{T2} = P_{2,set} - P_3$$

and its pressure ratio is:

(242) $$\Pi_2 = \frac{P_{2,set}}{P_3}$$

For the second turbine to produce positive expansion work:

(243) $$P_{2,set} > P_3$$

which requires:

(244) $$F_P > 1 + \frac{\Delta P_{exit}}{P_a}$$

If a minimum useful second-stage pressure drop $\Delta P_{T2,min}$ is imposed, the minimum pressure factor is:

(245) $$F_{P,min} = 1 + \frac{\Delta P_{exit} + \Delta P_{T2,min}}{P_a}$$

Let P_{1e} be the pressure at the first-turbine outlet immediately before the principal spray-mixing region. It is related to Zone 2 pressure by:

(246) $$P_{1e} = P_{2,set} + \Delta P_{mix} + \Delta P_{12,duct}$$

The first-turbine pressure ratio is:

(247) $$\Pi_1 = \frac{P_1}{P_{1e}}$$

and positive first-stage expansion requires:

(248) $$P_1 > P_{1e}$$

The maximum feasible pressure factor allowed by the source pressure is therefore bounded by:

(249) $$F_P < \frac{P_1 - \Delta P_{mix} - \Delta P_{12,duct}}{P_a}$$

The pressure factor must lie in the non-empty interval:

(250) $$F_{P,min} \leq F_P < F_{P,max}$$

where:

(251) $$F_{P,max} = \frac{P_1 - \Delta P_{mix} - \Delta P_{12,duct} - \Delta P_{T1,min}}{P_a}$$

The distributed and minor pressure losses used above are calculated from:

(252) $$\Delta P_{duct,k} = \left(f_k \frac{L_k}{D_{h,k}} + \sum_i K_{i,k} \right) \frac{\rho_k U_k^2}{2}$$

38. Forward-performance mode: mass flow from a specified inlet

When the inlet area A_1 is specified, the dry-air mass-flow rate is:

(253) $$\dot{m}_{da} = C_m \rho_{da,1} A_1 U_1$$

where C_m is a flow-capture or mass-flow coefficient accounting for inlet non-uniformity, contraction and bypass.

The incoming vapour mass-flow rate is:

(254) $$\dot{m}_{v,1} = \omega_1 \dot{m}_{da}$$

The total incoming humid-air mass-flow rate is:

(255) $$\dot{m}_1 = (1 + \omega_1) \dot{m}_{da}$$

and the inlet volumetric-flow rate is:

(256) $$\dot{V}_1 = A_1 U_1 = \frac{\dot{m}_1}{\rho_1}$$

For a circular inlet, the inlet diameter is:

(257) $$D_1 = \sqrt{\frac{4A_1}{\pi}}$$

39. First-turbine thermodynamic and mechanical output

The ideal isentropic outlet temperature of turbine 1 is:

(258) $$T_{1e,s} = T_1 \left(\frac{P_{1e}}{P_1} \right)^{(\gamma_1 - 1)/\gamma_1}$$

The actual outlet temperature is:

(259) $$T_{1e} = T_1 - \eta_{is,1} (T_1 - T_{1e,s})$$

The specific first-stage work, including a change in mean kinetic energy, is:

(260) $$w_{T1} = c_{p,m,1} (T_1 - T_{1e}) + \frac{U_1^2 - U_{1e}^2}{2}$$

The first-stage shaft power is:

(261) $$\dot{W}_{sh,1} = \eta_{m,1} \dot{m}_1 w_{T1}$$

The first generator electrical output is:

(262) $$\dot{W}_{el,1} = \eta_{g,1} \dot{W}_{sh,1}$$

These pressure-ratio, temperature-ratio and enthalpy-drop relationships are the standard first-order turbine calculation and should ultimately be replaced by a manufacturer turbine map.[15]

40. Coupled Zone 2 cooling, humidity, pressure and injection equations

The Zone 2 pressure is imposed by the control target:

(263) $$P_2 = P_{2,set} = F_P P_a$$

Let $\dot{m}_{evap}$ be the liquid mass flow that evaporates and $\dot{m}_c$ be the unevaporated carryover. The liquid balance is:

(264) $$\dot{m}_l = \dot{m}_{evap} + \dot{m}_c + \dot{m}_{drain}$$

The Zone 2 humidity ratio is:

(265) $$\omega_2 = \omega_1 + \frac{\dot{m}_{evap}}{\dot{m}_{da}}$$

The saturation humidity ratio at Zone 2 conditions is:

(266) $$\omega_{sat,2} = \varepsilon \frac{p_{ws}(T_2)}{P_2 - p_{ws}(T_2)}$$

The no-supersaturation constraint is:

(267) $$\omega_2 \le \omega_{sat,2}$$

The maximum thermodynamically absorbable evaporated-water flow is:

(268) $$\dot{m}_{evap,max} = \dot{m}_{da} max(\omega_{sat,2} - \omega_1, 0)$$

The total gas-phase flow entering turbine 2 is:

(269) $$\dot{m}_2 = \dot{m}_1 + \dot{m}_{evap}$$

A steady adiabatic energy balance across the spray zone is:

(270) $$\dot{m}_{da} h_{ma}(T_{1e}, \omega_1) + \dot{m}_l h_l(T_l) = \dot{m}_{da} h_{ma}(T_2, \omega_2) + (\dot{m}_l - \dot{m}_{evap}) h_l(T_2) + \dot{Q}_{loss,2}$$

where the drain and carryover streams leave at different temperatures, replace the combined downstream-liquid term by $\dot{m}_{carry} h_l(T_{carry}) + \dot{m}_{drain} h_l(T_{drain})$.

The moist-air enthalpy function used in this balance is:

(271) $$h_{ma}(T, \omega) = c_{p,da}(T - 273.15) + \omega\left[h_{fg,0} + c_{p,v}(T - 273.15)\right]$$

For a preliminary estimate with complete evaporation and small heat loss, the required evaporation rate can be approximated by:

(272a) $$\dot{m}_{evap,req} \approx \frac{\dot{m}_1 c_{p,m,1}(T_{1e} - T_2)}{h_v(T_2) - h_l(T_l)}$$

For a simplified liquid-water approximation:

(272b) $$h_v(T_2) - h_l(T_l) \approx h_{fg}(T_2) + c_{p,l}(T_2 - T_l)$$

Do not use $c_{p,v}$ for heating the injected liquid from T_l to T_2.

The estimate is feasible only if:

(273) $$0 \le \dot{m}_{evap,req} \le \dot{m}_{evap,max}$$

[15] NASA Glenn Research Center (2021) 'Power turbine thermodynamics'. Cleveland, OH: National Aeronautics and Space Administration. Available at: https://www.grc.nasa.gov/www/k-12/airplane/powtrbth.html (Accessed: 26 June 2026).

The Zone 2 mixture gas constant is:

(274) $$R_{m,2} = \frac{R_{da} + \omega_2 R_v}{1 + \omega_2}$$

and the Zone 2 gas density is:

(275) $$\rho_2 = \frac{P_2}{R_{m,2} T_2}$$

For a selected Zone 2 flow area A_2, continuity gives:

(276) $$U_2 = \frac{\dot{m}_2}{\rho_2 A_2}$$

The one-dimensional axial momentum balance through the spray and mixing region is:

(277) $$P_{1e} A_{12} + \dot{m}_1 U_{1e} + \dot{m}_l U_{l,x} = P_2 A_{12} + \dot{m}_2 U_2 + \dot{m}_{\text{carry}} U_{\text{carry,x}} + \dot{m}_{\text{drain}} U_{\text{drain}} + F_{wall} + F_{mix}$$

where:

(278) $$F_{wall} + F_{mix} = \left(f_2 \frac{L_2}{D_{h,2}} + \sum_i K_{i,2} \right) \frac{\rho_2 U_2^2}{2} A_{12}$$

The drain axial-velocity term may be set to zero only where the drain leaves perpendicular to the principal gas-flow direction and its axial momentum is negligible.

41. Evaporation kinetics and Zone 2 residence-time requirement

Let D_{32} be the Sauter mean droplet diameter. Under a D^2 evaporation approximation:

(279) $$\frac{\mathrm{d}D^2}{\mathrm{d}t} = -K_{evap}$$

The nominal complete-evaporation time is:

(280) $$t_{evap} = \frac{D_{32}^2}{K_{evap}}$$

The coefficient K_{evap} depends on gas temperature, pressure, humidity, relative velocity and heat and mass transfer. It should be determined from a validated spray model or experimental calibration.[16]

With evaporation safety factor $SF_{evap} > 1$, the required residence time is:

(281) $$t_{res,2} \geq SF_{evap} t_{evap}$$

The physical residence-time requirement is governed by evaporation and mixing. Sensor, controller and actuator time constants shall be assessed separately in the closed-loop dynamic model. They shall not be used directly to size the chamber unless the chamber is deliberately designed as a dynamic process buffer. More generally, the chamber residence time must exceed the largest relevant mixing, evaporation and control-response timescale:

(282) $$t_{res,2} \geq SF_t \max\left(t_{evap}, t_{mix}\right)$$

The Zone 2 volumetric flow is:

(283) $$\dot{V}_2 = \frac{\dot{m}_2}{\rho_2}$$

The required Zone 2 volume is:

[16] Sirignano, W.A. (2010) *Fluid Dynamics and Transport of Droplets and Sprays*. 2nd edn. Cambridge: Cambridge University Press.

(284) $V_{Z2} = \dot{V}_2 t_{res,2}$

and, for constant flow area A_2, its required axial length is:

(285) $L_{Z2} = \frac{V_{Z2}}{A_2} = U_2 t_{res,2}$

The actual evaporated fraction is approximated by:

(286) $\chi_{evap} = 1 - \left[\max\left(1 - \frac{K_{evap} t_{res,2}}{D_{32}^2}, 0\right)\right]^{3/2}$

The injected-liquid rate required to supply the calculated evaporated flow is:

(287a) $\dot{m}_l = \frac{\dot{m}_{evap}}{\chi_{evap}}$

subject to:

(287b) $\chi_{evap} \geq \chi_{evap,\min} > 0$

and:

(288) $\dot{m}_{carry} = \max\left[(1 - \chi_{evap})\dot{m}_l - \dot{m}_{drain}, 0\right)$

42. Second-turbine electrical output

The effective Zone 2 mixture heat capacity is:

(289) $c_{p,m,2} = \frac{c_{p,da} + \omega_2 c_{p,v}}{1 + \omega_2}$

and:

(290) $\gamma_2 = \frac{c_{p,m,2}}{c_{p,m,2} - R_{m,2}}$

The ideal isentropic turbine 2 outlet temperature is:

(291) $T_{3,s} = T_2 \left(\frac{P_3}{P_2}\right)^{(\gamma_2 - 1)/\gamma_2}$

The actual turbine 2 outlet temperature is:

(292) $T_3 = T_2 - \eta_{is,2}(T_2 - T_{3,s})$

The second-stage specific work is:

(293) $w_{T2} = c_{p,m,2}(T_2 - T_3) + \frac{U_2^2 - U_3^2}{2}$

The second-stage shaft power is:

(294) $\dot{W}_{sh,2} = \eta_{m,2} \dot{m}_2 w_{T2}$

and its electrical output is:

(295) $\dot{W}_{el,2} = \eta_{g,2} \dot{W}_{sh,2}$

43. Pump, control and net electrical energy output

The injector pressure must satisfy:

(296) $$\Delta P_{nozzle,in} \geq P_2 + \Delta P_{nozzle}$$

The pump pressure rise is:

(297) $$\Delta P_p \geq P_{pump,out} - P_{reservoir} + \Delta P_{suction}$$

The pump electrical demand is:

(298) $$\dot{W}_p = \frac{\dot{m}_l \Delta P_p}{\rho_l \eta_p \eta_{motor,p}}$$

The total auxiliary electrical demand is:

(299) $$\dot{W}_{aux} = \dot{W}_{controller} + \dot{W}_{sensors} + \dot{W}_{valves} + \dot{W}_{lubrication} + \dot{W}_{misc}$$

The gross electrical output is:

(300) $$\dot{W}_{el,gross} = \dot{W}_{el,1} + \dot{W}_{el,2}$$

The net electrical power is:

(301) $$\dot{W}_{el,net} = \dot{W}_{el,1} + \dot{W}_{el,2} - \dot{W}_p - \dot{W}_{aux}$$

Bearing, windage and generator losses shall not be subtracted separately if they are already incorporated in the mechanical and generator efficiencies. Each loss shall be counted once only.

The electrical energy generated over a variable operating period is:

(302) $$E_{el,net} = \int_0^{t_{op}} \dot{W}_{el,net}(t)\mathrm{d}t$$

For steady operation:

(303) $$E_{el,net} = \dot{W}_{el,net} t_{op}$$

The net specific electrical work per kilogram of dry air is:

(304) $$w_{el,net,da} = \frac{\dot{W}_{el,net}}{\dot{m}_{da}}$$

A necessary economic and energetic feasibility condition is:

(305) $$\dot{W}_{el,net} > 0$$

44. Inverse-sizing mode from a required electricity output

For a specified target electrical energy and operating duration, first calculate:

(306) $$\dot{W}_{el,target} = \frac{E_{el,target}}{t_{op}}$$

At a trial unit dry-air mass flow, solve the coupled thermodynamic equations and calculate the net specific electrical work:

(307) $$w_{el,net,da} = w_{el,1,da} + w_{el,2,da} - w_{aux,da}$$

If $w_{el,net,da} > 0$, the required dry-air mass flow is:

(308) $$\dot{m}_{da,req} = \frac{\dot{W}_{el,target}}{w_{el,net,da}}$$

The required inlet area is:

(309) $$A_{1,req} = \frac{\dot{m}_{da,req}}{C_m \rho_{da,1} U_1}$$

and the required circular inlet diameter is:

(310) $$D_{1,req} = \sqrt{\frac{4\dot{m}_{da,req}}{\pi C_m \rho_{da,1} U_1}}$$

Because pressure losses, turbine efficiencies, Reynolds number, droplet residence time and parasitic power change with scale, the preceding calculation is iterative. Denoting the complete sizing operator by S:

(311) $$d^{(n+1)} = S\left(z_{in}, \dot{W}_{el,target}, d^{(n)}\right)$$

Convergence is obtained when:

(312) $$\frac{\left|\dot{W}_{el,net}^{(n)} - \dot{W}_{el,target}\right)}{\dot{W}_{el,target}} \le \varepsilon_W$$

45. Duct and turbine dimensional requirements

Select allowable mean axial velocities $U_{k,des}$ for each station. The required flow areas are:

(313) $$A_k = \frac{\dot{m}_k}{\rho_k U_{k,des}}$$

and equivalent circular duct diameters are:

(314) $$D_k = \sqrt{\frac{4A_k}{\pi}}$$

For turbine j, the required rotor annulus area is:

(315) $$A_{ann,j} = \frac{\dot{m}_j}{\rho_j U_{ax,j}}$$

Let the hub-to-tip diameter ratio be $\lambda_{h,j} = D_{hub,j} / D_{tip,j}$. Then:

(316) $$D_{tip,j} = \sqrt{\frac{4A_{ann,j}}{\pi\left(1-\lambda_{h,j}^2\right)}}$$

and:

(317) $$D_{hub,j} = \lambda_{h,j} D_{tip,j}$$

If the selected blade-speed ratio is $\lambda_{u,j} = U_{tip,j} / U_{ax,j}$, the shaft speed is:

(318) $$N_j = \frac{60\lambda_{u,j} U_{ax,j}}{\pi D_{tip,j}}$$

The angular velocity is:

(319) $$\Omega_j = \frac{2\pi N_j}{60}$$

and the turbine shaft torque is:

(320) $$\tau_j = \frac{\dot{W}_{sh,j}}{\Omega_j}$$

A preliminary solid circular shaft diameter based on allowable torsional shear stress is:

(321) $$d_{s,j} \geq \left(\frac{16 SF_{s,j} \tau_j}{\pi \tau_{allow,j}} \right)^{1/3}$$

For a thin cylindrical duct subject to maximum internal differential pressure ΔP_{max}, a preliminary wall thickness is:

(322) $$t_w \geq \frac{SF_w \Delta P_{max} D}{2 \eta_w \sigma_{allow,w}}$$

This wall-thickness expression is only a preliminary membrane-stress estimate; final structural design must include buckling, openings, weld efficiency, fatigue, vibration, corrosion allowance and the applicable pressure-vessel or duct standard.

The flow-conditioning lengths may be parameterised as:

(323) $$L_{in} = n_{in} D_1$$

(324) $$L_{out} = n_{out} D_3$$

The complete principal system length is:

(325) $$L_{system} = L_{in} + L_{T1} + L_{Z2} + L_{T2} + L_{out}$$

46. Injector, liquid line, reservoir and pump sizing

For an injector discharge coefficient C_d, the total required nozzle-orifice area is:

(326) $$A_{or,total} = \frac{\dot{m}_l}{C_d \sqrt{2 \rho_l \left(P_{nozzle,in} - P_2 \right)}}$$

For n_{or} equal circular orifices, the individual orifice diameter is:

(327) $$d_{or} = \sqrt{\frac{4 A_{or,total}}{\pi n_{or}}}$$

The selected atomiser must satisfy the droplet-size requirement:

(328) $$D_{32} = F_{nozzle}\left(d_{or}, \Delta P_{nozzle}, \rho_l, \mu_l, \sigma_l, \rho_2, U_{rel} \right) \leq D_{32,max}$$

The minimum liquid-line area for an allowable line velocity $U_{l,max}$ is:

(329) $$A_l \geq \frac{\dot{m}_l}{\rho_l U_{l,max}}$$

and the corresponding line diameter is:

(330) $$D_l \geq \sqrt{\frac{4 \dot{m}_l}{\pi \rho_l U_{l,max}}}$$

The reservoir volume required for an unattended operating period t_{resv} is:

(331) $$V_{reservoir} \geq SF_{resv} \frac{\dot{m}_l t_{resv}}{\rho_l}$$

The pump rated flow and power are:

(332) $$\dot{m}_{p,rated} \geq SF_{p,Q}\dot{m}_l$$

(333) $$\dot{W}_{p,rated} \geq SF_{p,W}\dot{W}_p$$

47. Generator and electrical-system ratings

The rated generator powers should satisfy:

(334) $$\dot{W}_{g,j,rated} \geq SF_{g,j}\dot{W}_{el,j,max}$$

The rated generator torque must satisfy:

(335) $$\tau_{g,j,rated} \geq SF_{\tau,j}\frac{\dot{W}_{sh,j,max}}{\Omega_{j,min}}$$

For a three-phase generator with line voltage V_L, power factor PF and conversion efficiency η_{conv}, the rated line current is:

(336) $$I_{L,rated} \geq \frac{\dot{W}_{el,rated}}{\sqrt{3}V_L PF}$$

where the calculation begins with rated shaft power, first calculate $\dot{W}_{el,rated} = \eta_{conv}\dot{W}_{sh,rated}$. Conversion efficiency shall not then be divided into the electrical line-current equation a second time.

The rated apparent power is:

(337) $$S_{rated} \geq \frac{\dot{W}_{el,rated}}{PF}$$

Generator selection, temperature rise, duty, insulation, overspeed and rating verification should comply with the applicable rotating-machine requirements, including IEC 60034-1:2026 where applicable.[17]

48. Complete simultaneous design-and-performance equation system

Define the principal unknown vector as:

(338) $$y = \left[\dot{m}_{da} \quad A_1 \quad P_{1e} \quad T_{1e} \quad P_2 \quad T_2 \quad \omega_2 \quad \dot{m}_{evap} \quad \dot{m}_l \quad \rho_2 \quad U_2 \quad P_3 \quad T_3 \quad \dot{W}_{el,net} \quad A_2\right]^T$$

For forward mode, A_1 is prescribed and $\dot{W}_{el,net}$ is an output. For inverse mode, $\dot{W}_{el,net} = \dot{W}_{el,target}$ is prescribed and A_1 is an output.

The following residual system defines the preliminary coupled thermodynamic and sizing calculation. It is closed only after one geometric relation for A_2, one evaporation-kinetics relation and the applicable forward- or inverse-mode closure have been included. The closed calculation is represented by:

(339) $$F\left(y; z_{in}, p\right) = 0$$

where p contains efficiencies, loss coefficients, fluid properties, droplet-model coefficients and selected design ratios. An explicit residual form is:

[17] International Electrotechnical Commission (2026) *IEC 60034-1:2026 Rotating electrical machines, Part 1: Rating and performance*. Geneva: IEC.

$$F_1 = P_2 - F_P P_a = 0$$

$$F_2 = P_3 - P_a - \Delta P_{exit} = 0$$

$$F_3 = P_{1e} - P_2 - \Delta P_{mix} - \Delta P_{12,duct} = 0$$

$$F_4 = T_{1e} - T_1 + \eta_{is,1}\left(T_1 - T_{1e,s}\right) = 0$$

$$F_5 = \omega_2 - \omega_1 - \dot{m}_{evap} / \dot{m}_{da} = 0$$

$$F_6 = E_{spray}\left(T_2, \omega_2, \dot{m}_l, \dot{m}_{evap}\right) = 0$$

$$F_7 = M_{spray}\left(P_{1e}, P_2, U_{1e}, U_2, \dot{m}_l\right) = 0$$

$$F_8 = \rho_2 - P_2 / \left(R_{m,2} T_2\right) = 0$$

$$F_9 = U_2 - \dot{m}_2 / \left(\rho_2 A_2\right) = 0$$

$$F_{10} = T_3 - T_2 + \eta_{is,2}\left(T_2 - T_{3,s}\right) = 0$$

$$F_{11} = \dot{m}_{\mathrm{evap}} - \chi_{\mathrm{evap}} \dot{m}_l = 0$$

(340)

$$F_{12} = \dot{W}_{el,net} - \dot{W}_{el,1} - \dot{W}_{el,2} + \dot{W}_p + \dot{W}_{aux} = 0$$

A fixed area-ratio or equivalent geometric closure is:

(341a) $$F_{13} = A_2 - r_{\mathrm{A}} A_1 = 0$$

where $r_A = A_2 / A_1$ is a selected design parameter.

The inlet mass-flow closure in both modes is:

(341b) $$F_{14} = \dot{m}_{da} - C_m \rho_{da,1} A_1 U_1 = 0$$

Forward mode adds the closure equation:

(342) $$F_{15}^{(forward)} = A_1 - A_{1,given} = 0$$

whereas inverse mode adds:

(343) $$F_{15}^{(inverse)} = \dot{W}_{el,net} - \dot{W}_{el,target} = 0$$

The solution now contains 15 unknowns and 15 residual equations in either forward or inverse mode. Turbine-map constraints must still be checked after each solve or incorporated as additional residuals with corresponding additional unknowns.

The simultaneous equations should be solved with a bounded nonlinear solver, followed by turbine-map, CFD and structural verification. Gas-turbine acceptance testing and performance correction should be based on an appropriate test code such as ISO 2314, modified as necessary for this non-combustion, liquid-injected device.[18]

49. Mandatory feasibility and design constraints

The atmospheric and inlet humidity inputs must satisfy:

(344) $$0 \le \phi_1 \le 1, 0 \le \phi_a \le 1$$

The incoming pressure must provide sufficient head for both stages and all losses:

(345) $$P_1 > P_{2,set} + \Delta P_{T1,min} + \Delta P_{mix} + \Delta P_{12,duct}$$

The Zone 2 pressure-factor constraint is:

[18] International Organization for Standardization (2009) *ISO 2314:2009 Gas turbines—Acceptance tests*. 3rd edn. Geneva: ISO.

(346) $P_2 \geq F_P P_a > P_a$

The second-stage expansion constraint is:

(347) $P_2 > P_3 \geq P_a$

The available evaporation capacity must exceed the required evaporation:

(348) $\dot{m}_{evap,req} \leq \dot{m}_{evap,max}$

The carryover fraction must satisfy:

(349) $\frac{\dot{m}_c}{\dot{m}_l} \leq f_{carry,max}$

The flow must remain within prescribed compressibility limits:

(350) $Ma_k = \frac{U_k}{\sqrt{\gamma_k R_{m,k} T_k}} \leq Ma_{max}$

The duct Reynolds number is:

(351) $Re_k = \frac{\rho_k U_k D_{h,k}}{\mu_k}$

and must lie within the range for which the selected friction, heat-transfer and turbine correlations are valid.

The turbine operating points must remain inside their calibrated maps:

(352) $\left(\dot{m}_{corr,j}, N_{corr,j}, \Pi_j\right) \in T_j$

The rotor tip-speed constraints are:

(353) $\Omega_j \frac{D_{tip,j}}{2} \leq U_{tip,j,max}$

The structural stress constraints are:

(354) $\sigma_{eq,j} \leq \frac{\sigma_{yield,j}}{SF_j}$

The generator and pump ratings must satisfy:

(355) $\dot{W}_{g,j,rated} \geq \dot{W}_{el,j,max}, \dot{W}_{p,rated} \geq \dot{W}_{p,max}$

The complete design is feasible only when:

(356) $C\left(y, z_{in}, p\right) \leq 0$

and:

(357) $\dot{W}_{el,net} > 0$

50. Calculation sequence and required output schedule

A practical calculation proceeds as follows:

1. Read U_1, ϕ_1, P_1, T_1, P_a, T_a, ϕ_a, F_P and the scale-setting input.
1. Calculate vapour partial pressures, humidity ratios, densities, enthalpies, specific heats and gas constants.
2. Set $P_2 = F_P P_a$ and calculate the allowable pressure ratios after duct and mixing losses.
3. Calculate the first-stage outlet state and first-stage shaft and electrical power.
4. Solve the coupled spray-zone energy, mass, saturation, state, continuity and momentum equations.

5. Determine liquid flow, droplet size, residence time, Zone 2 volume and length.
6. Calculate the second-stage outlet state and second-stage shaft and electrical power.
7. Calculate pump and auxiliary demands and net electrical power and energy.
8. In inverse mode, iterate mass flow and area until the requested power or energy is achieved.
9. Size duct diameters, turbine annuli, rotor diameters, shaft diameters, injectors, piping, reservoir, pump and generators.
10. Reject any solution violating pressure-factor, saturation, carryover, turbine-map, Mach-number, structural or electrical constraints.

The final calculation schedule should report at least:

- all input conditions and units;
- $P_{2,set}$ and the selected pressure margin above atmosphere;
- dry-air, vapour, total-gas and injected-liquid mass flows;
- all station pressures, temperatures, humidity ratios, densities and velocities;
- turbine pressure ratios, efficiencies, speeds, torques and shaft powers;
- generator gross outputs, pump and auxiliary demands, net power and generated energy;
- inlet, duct, Zone 2, turbine, shaft, nozzle, liquid-line and reservoir dimensions;
- evaporation fraction, residence time, carryover and saturation margin;
- all active design constraints and safety factors; and
- sensitivity of net power and dimensions to inlet velocity, humidity, pressure, temperature and F_P .

The patent provides the architecture and control concept but not the numerical component characteristics needed to validate a particular design.[19]

[19] de St. Amatus, B. and de St. Amatus, A. (2026) *Device for conversion of kinetic and thermal energies of gas flows to work*. US Patent 12,650,086 B1, 9 June.

51. Constrained energy-optimising regulator, real I/O implementation and recommended hardware

This section converts the preceding thermodynamic and sizing model into a regulator that can be implemented on an industrial programmable controller. The regulator has three deliberately separated layers: a fast pressure-protection loop, a slower net-power optimisation loop, and an independent protective trip layer. The separation is essential because optimisation must never be permitted to override the minimum Zone 2 pressure, rotor-speed, temperature, vibration, carryover or electrical limits. The controller architecture follows the patent's requirement for pressure and temperature sensing in the internal zones and atmosphere and for real-time regulation of the evaporating-liquid injector.[20]

The sample program is an engineering implementation template. It is **not** a certified safety program, a final turbine governor or a substitute for a hazard and operability study, layer-of-protection analysis, safety-requirements specification, machinery risk assessment, independent overspeed protection or validated emergency shutdown system. Any production implementation must be reviewed and commissioned by competent process, electrical, rotating-equipment, functional-safety and control engineers.

51.1 Control objective and pressure-factor boundary

Let the user-selected pressure factor be $F_P > 1$. The normal Zone 2 pressure setpoint is:

(358) $$P_{2,set} = F_P P_a$$

The protective trip factor F_{trip} must be lower than the normal factor but remain above unity:

(359) $$P_{2,trip} = F_{trip} P_a, 1 < F_{trip} < F_P$$

A useful dimensionless pressure reserve is:

(360) $$m_P = \frac{P_2 - P_{2,set}}{P_a}$$

Positive m_P means the measured Zone 2 pressure remains above the normal pressure-factor target. The optimisation layer is suspended before m_P reaches zero; the fast loop then reduces liquid injection and coordinates generator loading to restore pressure reserve.

The complete measured-input vector used by the regulator is:

(361) $$y = \left[P_1, P_{2A}, P_{2B}, P_{2C}, P_a, T_1, T_2, T_a, \phi_1, \phi_a, U_1, N_1, N_2, P_{el,1}, P_{el,2}, P_{pump}, P_{aux}\right)^T$$

Three Zone 2 pressure channels are shown because pressure immediately upstream of the second turbine is the principal protective variable. A lower-integrity laboratory rig may begin with one transmitter, but a production design should use redundancy appropriate to the risk assessment.

51.2 Real-sensor acquisition, scaling and validation

The preferred field signal for ordinary industrial transmitters is isolated 4–20 mA, with HART, PROFINET, EtherCAT, Modbus TCP, PROFIBUS PA or another deterministic fieldbus used where diagnostics are required. For a raw analogue input r whose configured digital span is r_{min} to r_{max}, the engineering value is:

(362) $$x = x_{min} + \frac{r - r_{min}}{r_{max} - r_{min}}\left(x_{max} - x_{min}\right)$$

A measurement channel is valid only when its raw current, module status, rate of change and engineering range are acceptable:

(363) $$v_i = 1\left[r_{lo} \le r_i \le r_{hi}\right)1\left[\left|\frac{dx_i}{dt}\right) \le R_{i,max}\right)1\left[x_{i,min} \le x_i \le x_{i,max}\right)1\left[d_i = 0\right)$$

---

[20] de St. Amatus, B. and de St. Amatus, A. (2026) *Device for conversion of kinetic and thermal energies of gas flows to work*. US Patent 12,650,086 B1, 9 June.

where d_i is the channel diagnostic word. A first-order discrete filter suppresses electrical noise without concealing a genuine rapid pressure loss:

(364) $$x_k = x_{k-1} + \alpha_x\left(x_k - x_{k-1}\right), \alpha_x = \frac{\Delta t}{\tau_x + \Delta t}$$

For three valid Zone 2 pressure channels, the voted pressure is:

(365) $$P_2 = median\left(P_{2A}, P_{2B}, P_{2C}\right)$$

A pairwise disagreement alarm should be raised whenever the maximum channel spread exceeds the commissioned tolerance. No optimisation step is accepted while a required channel is invalid, stale or disagreeing.

51.3 Online power and energy calculation

The measured net electrical power is calculated from the two generator outputs and all measured parasitic loads:

(366) $$P_{net} = P_{el,1} + P_{el,2} - P_{pump} - P_{aux}$$

Generated net electrical energy is accumulated numerically:

(367) $$E_{net,k} = E_{net,k\text{-}1} + \frac{\Delta t}{3600} P_{net,k}$$

where P_{net} is expressed in kilowatts, Δt is in seconds and E_{net} is in kilowatt-hours. The controller should retain both the instantaneous value and rolling averages over several time horizons so that the slow optimiser does not respond to measurement noise or transient storage of energy in rotating inertia.

For every inequality constraint $g_j(x,u) \leq 0$, define the non-negative reserve:

(368) $$s_j = \text{-}g_j(x,u)$$

The operating objective is to maximise measured net power while preserving all reserves. An equivalent minimisation objective is:

(369) $$J = \text{-}P_{net} + \sum_j w_j \left[max\left(0, -s_j\right)\right]^2 + w_{\Delta u} \|\Delta u\|_2^2$$

The first penalty prevents a candidate operating point from being accepted after a constraint violation; the second limits unnecessary actuator movement and hunting.

51.4 Fast pressure loop and coordinated generator governor

Define the signed pressure error so that positive error permits additional liquid injection and negative error requires injection to decrease:

(370) $$e_P = P_2 - P_{2,set}$$

The unconstrained pressure-controller output is:

(371) $$u_{P,raw} = K_P e_P + K_I z_P + K_D \frac{de_P}{dt}$$

An anti-windup integrator is used because the injection valve and pump have finite ranges:

(372) $$\dot{z}_P = e_P + K_{aw}\left(u_{P,sat} - u_{P,raw}\right)$$

The commanded liquid flow combines the model-based feedforward estimate, the pressure controller and the slow optimiser's bias:

(373a) $$u_{valve,cmd} = \text{clip}\left(u_{valve,ff} + u_{P,sat} + b_l, 0, 100\right)$$

(373b) $$\dot{m}_l = f_{valve}\left(u_{valve,cmd}, u_{pump,cmd}, P_{nozzle,in}, P_2\right)$$

The generator torque or electrical-load command for turbine j is coordinated with rotor speed and the optimiser:

(374) $$\tau_{g,j,cmd} = \mathrm{clip}\left(\tau_{g,j,ff} + K_{N,j}\left(N_j - N_{j,set}\right) + b_{g,j}, \tau_{g,j,min}, \tau_{g,j,max}\right)$$

All analogue commands are rate-limited:

(375) $$u_k = \mathrm{clip}\left(u_k, u_{k\text{-}1} - R_u \Delta t, u_{k\text{-}1} + R_u \Delta t\right)$$

The sign and gain of the generator governor must be confirmed from the selected turbine-generator topology. Removing generator load may accelerate a turbine and therefore is not automatically a safe response. The trip value must be a pre-validated braking, dump-load, converter or bypass condition for the actual machine.

When the protective layer is active, the command vector is replaced by the commissioned safe vector:

(376) $$u = u_{safe} = \left[0, 0, \tau_{g,1,safe}, \tau_{g,2,safe}, u_{bypass,safe}\right)^T$$

The first two elements close the liquid valve and stop the injection pump. The generator and bypass elements are installation-specific and must be established through overspeed, pressure-transient and loss-of-load analysis.

51.5 Slow constrained power optimiser

The slow optimiser modifies only bounded biases around the stable fast-loop operating point:

(377) $$\theta = \left[b_l, b_{g,1}, b_{g,2}\right)^T$$

A commissioning-safe coordinate perturbation estimates the local power gradient for decision variable i:

(378) $$g_i = \frac{P_{net}\left(\theta + \delta_i e_i\right) - P_{net}\left(\theta - \delta_i e_i\right)}{2\delta_i}$$

Where symmetric tests would approach a constraint, the program uses a one-sided perturbation. A pressure-reserve barrier and penalties for approaching other limits can be added to the measured objective:

(379) $$\Phi(\theta) = P_{net} - \lambda_P\left[\max\left(0, m_{P,min} - m_P\right)\right)^2 - \sum_{j \neq P} \lambda_j \left[\max\left(0, s_{j,min} - s_j\right)\right)^2$$

The accepted update is projected into the feasible command set:

(380) $$\theta_{k+1} = \Pi_\Omega\left[\theta_k + \alpha_k \nabla \Phi_k\right)$$

where:

(381) $$\Omega = \left\{\theta : P_2 \geq P_{2,trip}, N_j \leq N_{j,max}, T_k \in \left[T_{k,min}, T_{k,max}\right), \dot{m}_l \in \left[\dot{m}_{l,min}, \dot{m}_{l,max}\right), g_{other} \leq 0\right)$$

For initial commissioning, a perturb-and-observe coordinate search is preferable to an aggressive model-predictive optimiser because its individual decisions are visible, reversible and readily bounded. After validated turbine maps, spray-zone models and actuator dynamics are available, the same architecture may use constrained model-predictive control.

51.6 Trip, latching and restart conditions

The software trip request is:

(382) $$q_{trip} = q_{E\text{-}stop} \vee \neg q_{IO} \vee \neg q_{sensor} \vee \left(P_2 \leq P_{2,trip}\right) \vee \ _j \left(N_j \geq N_{j,trip}\right) \vee q_T \vee q_{vib} \vee q_{carry} \vee q_{drive}$$

The software request must be combined with an independent hardwired or safety-rated shutdown path. A restart is permitted only when all causes are cleared, the operator has issued a deliberate reset and a stable dwell period has elapsed:

(383) $$q_{restart} = q_{reset} \wedge \neg q_{trip} \wedge \left(P_2 > P_{2,set} + \Delta P_{restart}\right) \wedge \left(t_{stable} \geq t_{restart,min}\right)$$

An automatic restart after an emergency stop, overspeed, loss of Zone 2 pressure or loss of critical instrumentation is not recommended.

51.7 Regulator algorithm

The following sequence is executed cyclically:

1. Read physical analogue inputs, fieldbus values, channel status words, digital interlocks and watchdogs.
2. Scale currents to engineering units and reject underrange, overrange, stale, implausible or rapidly discontinuous values.
3. Median-vote the three Zone 2 pressure channels and raise a disagreement alarm when the spread exceeds tolerance.
4. Filter measurement noise using sensor-specific time constants short enough to retain protective response.
5. Calculate humidity ratios, mixture properties, pressure factors, net electrical power and accumulated energy.
6. Calculate $P_{2,set}$ and $P_{2,trip}$ from the current measured atmospheric pressure.
7. Evaluate hard trip conditions independently of the optimisation algorithm.
8. If a trip condition is present, latch the trip, close the injection valve, stop the pump, command the validated generator/bypass safe state, energise the alarm path as designed and suspend optimisation.
9. If operation is permitted, run the fast pressure PID and generator governor.
10. Project all commands through amplitude, rate, turbine-map, speed, temperature, vibration, carryover and electrical limits.
11. At the slower optimisation interval, perturb only one decision variable and wait for the defined settling period.
12. Accept the perturbation only when filtered net power improves and every constraint retains its minimum reserve; otherwise restore the previous command, reverse the search direction and reduce the step.
13. Write analogue and digital outputs and confirm valve, pump, drive and generator feedback.
14. Log measurements, commands, active constraints, alarm/trip causes, optimisation decisions, net power and energy.
15. Repeat the cyclic scan.

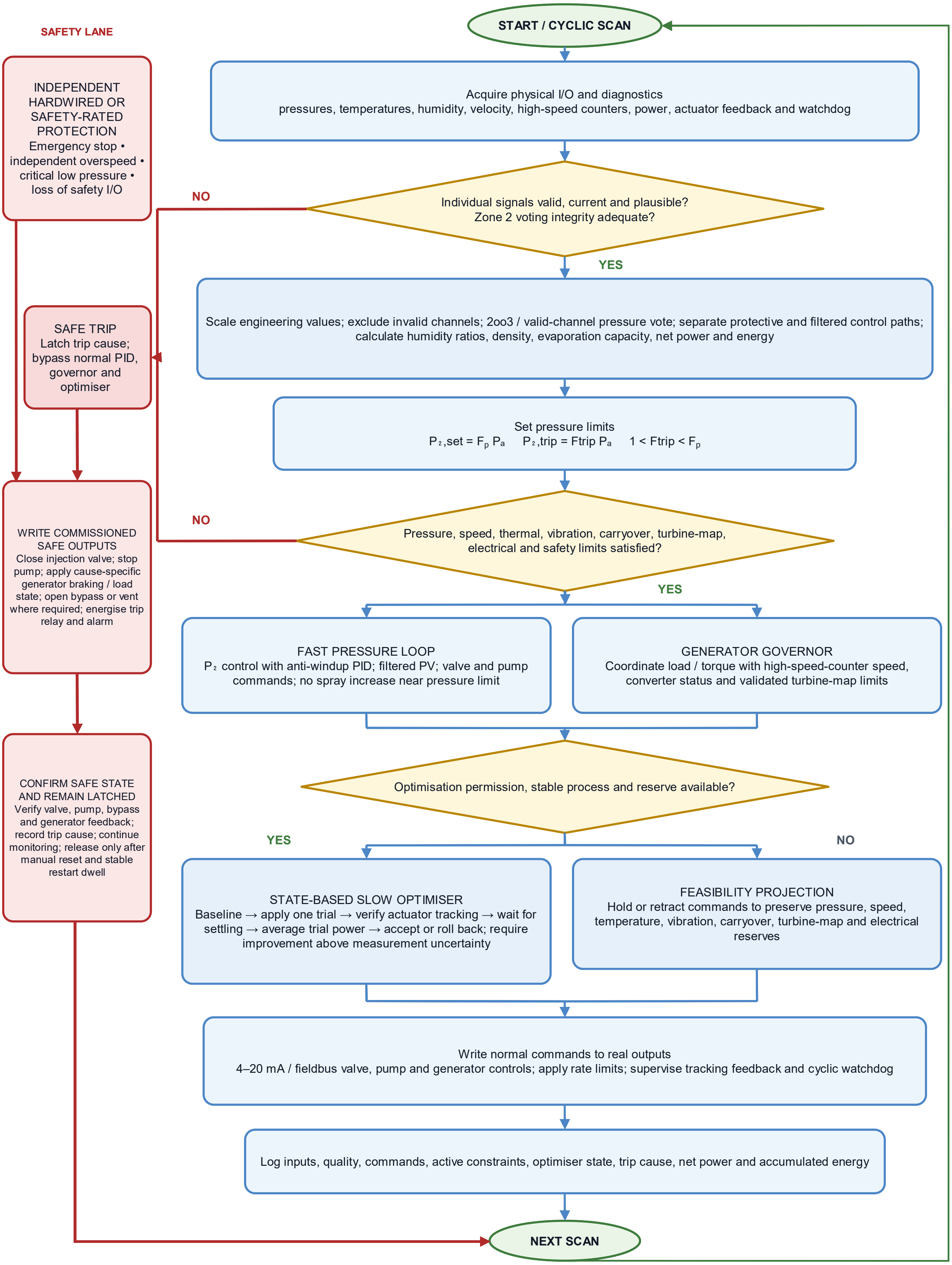

Figure 51.7. Hierarchical constrained energy-optimisation regulator and independent trip path.

51.8 Physical I/O allocation

A representative pilot-scale I/O allocation is shown below. Addresses are examples and must be replaced by the engineering addresses generated by the selected PLC or remote-I/O configuration.

Signal	Recommended physical interface	Example program tag	Protective treatment
Zone 1 absolute pressure	Isolated 4–20 mA or fieldbus	`AI_P1_RAW`	Range and rate validation
Zone 2 absolute pressure A/B/C	Three independent isolated 4–20 mA channels	`AI_P2A_RAW`, `AI_P2B_RAW`, `AI_P2C_RAW`	Median vote, spread alarm and trip
Atmospheric pressure	4–20 mA or digital ambient transmitter	`AI_PA_RAW`	Used in every pressure-factor calculation
Zone 1 and Zone 2 temperature	Pt100 transmitter, thermocouple module or 4–20 mA transmitter	`AI_T1_RAW`, `AI_T2_RAW`	Range, rate and open-sensor diagnostics
Ambient temperature and humidity	Digital transmitter or 4–20 mA	`AI_TA_RAW`, `AI_RHA_RAW`	Plausibility and stale-data check
Inlet humidity	Heated humidity probe or conditioned sample system	`AI_RH1_RAW`	Condensation protection and diagnostic status
Inlet velocity or mass flow	4–20 mA, pulse or industrial Ethernet	`AI_U1_RAW`	Range and flow-direction check
Turbine speeds	Redundant speed pickups to high-speed counter/safety system	`N1_RPM`, `N2_RPM`	Independent overspeed trip
Generator and parasitic powers	Power meter over Modbus TCP/PROFINET or analogue transducer	`PEL1_KW`, `PEL2_KW`, `PPUMP_KW`, `PAUX_KW`	Communication watchdog and value-quality bit
Liquid valve	Isolated 4–20 mA output with position feedback	`AO_VALVE_RAW`	Fail-closed unless the risk study determines otherwise
Injection pump	VSD or digital dosing-pump command plus run feedback	`AO_PUMP_RAW`, `DO_PUMP_ENABLE`	Stop on trip and prove stopped
Generator loading	Converter/VFD/dump-load setpoint plus torque/speed feedback	`AO_LOAD1_RAW`, `AO_LOAD2_RAW`	Pre-validated safe braking/load state
Bypass or vent	Safety-rated solenoid/actuator where required	`DO_BYPASS_SAFE`	Independent de-energise/energise philosophy from risk study

51.9 Sample IEC 61131-3 Structured Text regulator

The following sample uses IEC 61131-3 Structured Text and a Siemens-style raw analogue span of 0 to 27,648 for a configured 4–20 mA channel. It demonstrates physical I/O mapping, engineering-unit scaling, individual signal-quality and stale-data validation, valid-channel Zone 2 pressure voting, separate filtered control and minimally filtered protective pressure paths, high-speed-counter turbine-speed acquisition, pressure-factor regulation, psychrometric and evaporation-capacity calculations, actuator command writing, actuator-position and safe-state feedback, watchdog supervision, latched and cause-specific trip handling, restart dwell verification, and a bounded state-based perturb-and-observe optimiser. The program must be adapted to the selected processor, I/O modules, fieldbus and device-diagnostic architecture, calibrated signal ranges, electrical fail states, actuator dynamics, turbine maps, generator and converter arrangement, braking or dump-load system, and independently engineered safety architecture. It is an engineering template and must **not** be treated as a certified safety program, final turbine governor or production-ready control application.

```
(**********************************************************************
 SECTION 51.9
 SAMPLE IEC 61131-3 STRUCTURED TEXT REGULATOR

 ENGINEERING TEMPLATE — NOT A CERTIFIED SAFETY PROGRAM

 Nominal fast-loop period: 50 ms.
 Actual elapsed task time is used where available.

 Assumed process sign:
 Increasing liquid injection reduces Zone 2 pressure.

 IMPORTANT:
 1. Independent overspeed, emergency-stop and critical pressure
    protection must remain outside this optimisation program.
 2. All pressure measurements used in pressure ratios are absolute.
 3. Valve, pump and generator commands are percentages, not mass flow.
 4. Measured liquid flow is represented separately in kg/s.
 5. Safe generator commands are trip-cause-specific and must be
    established by machine and converter studies.
 6. HART, PROFINET and module diagnostic processing is performed by
    the configured device/communications layer and presented to this
    program as value, quality and stale-data signals.
**********************************************************************)


TYPE E_TripCause :
(
    TC_NONE,
    TC_EMERGENCY_STOP,
    TC_INDEPENDENT_SAFETY,
    TC_REMOTE_IO,
    TC_ZONE2_LOW_PRESSURE,
    TC_TURBINE1_OVERSPEED,
    TC_TURBINE2_OVERSPEED,
    TC_TEMPERATURE,
    TC_VIBRATION,
    TC_LIQUID_CARRYOVER,
    TC_DRIVE_OR_CONVERTER,
    TC_CRITICAL_SENSOR,
    TC_WATCHDOG
);
END_TYPE


TYPE E_OptimizerState :
(
    OPT_BASELINE,
    OPT_APPLY_TRIAL,
    OPT_WAIT_ACTUATOR,
    OPT_WAIT_SETTLE,
    OPT_AVERAGE_TRIAL,
    OPT_DECIDE
);
END_TYPE


VAR_GLOBAL
    (*
       Physical analogue inputs.
       Example Siemens-style process-image addresses.
       The final addresses depend on the selected I/O hardware.
    *)
    AI_P1_RAW          AT %IW0  : INT;
    AI_P2A_RAW         AT %IW2  : INT;
    AI_P2B_RAW         AT %IW4  : INT;
    AI_P2C_RAW         AT %IW6  : INT;
    AI_PA_RAW          AT %IW8  : INT;

    AI_T1_RAW          AT %IW10 : INT;
    AI_T2_RAW          AT %IW12 : INT;
    AI_TA_RAW          AT %IW14 : INT;

    AI_RH1_RAW         AT %IW16 : INT;
    AI_RH2_RAW         AT %IW18 : INT;
    AI_RHA_RAW         AT %IW20 : INT;

    AI_U1_RAW          AT %IW22 : INT;
```

```
AI_LIQFLOW_RAW     AT %IW24 : INT;
AI_VALVE_POS_RAW   AT %IW26 : INT;

AI_PEL1_RAW        AT %IW28 : INT;
AI_PEL2_RAW        AT %IW30 : INT;
AI_PPUMP_RAW       AT %IW32 : INT;
AI_PAUX_RAW        AT %IW34 : INT;

AI_VIB1_RAW        AT %IW36 : INT;
AI_VIB2_RAW        AT %IW38 : INT;
AI_CARRY_RAW       AT %IW40 : INT;

(*
   High-speed-counter or frequency-module values.
   Example unit: millihertz from the configured counter module.
   These values are for normal control and monitoring only.
   Independent overspeed protection remains separate.
*)
HSC_N1_FREQ_MHZ    AT %ID44 : DINT;
HSC_N2_FREQ_MHZ    AT %ID48 : DINT;

(* Physical analogue outputs. *)
AO_VALVE_RAW       AT %QW0 : INT;
AO_PUMP_RAW        AT %QW2 : INT;
AO_LOAD1_RAW       AT %QW4 : INT;
AO_LOAD2_RAW       AT %QW6 : INT;

(* Primary digital inputs. *)
DI_ESTOP_OK              AT %I0.0 : BOOL;
DI_REMOTE_IO_OK          AT %I0.1 : BOOL;
DI_PUMP_READY            AT %I0.2 : BOOL;
DI_DRIVE1_READY          AT %I0.3 : BOOL;
DI_DRIVE2_READY          AT %I0.4 : BOOL;
DI_RESET                 AT %I0.5 : BOOL;
DI_SAFETY_SYSTEM_OK      AT %I0.6 : BOOL;
DI_WATCHDOG_RETURN_OK    AT %I0.7 : BOOL;

(* Protective-system status inputs. *)
DI_TEMP_PROTECTION_OK    AT %I1.0 : BOOL;
DI_VIB_PROTECTION_OK     AT %I1.1 : BOOL;
DI_CARRY_PROTECTION_OK   AT %I1.2 : BOOL;
DI_TURBINE1_MAP_OK       AT %I1.3 : BOOL;
DI_TURBINE2_MAP_OK       AT %I1.4 : BOOL;
DI_ELECTRICAL_LIMITS_OK  AT %I1.5 : BOOL;
DI_INDEPENDENT_TRIP_OK   AT %I1.6 : BOOL;
DI_DRIVES_STABLE         AT %I1.7 : BOOL;

(* Final-element and tracking feedback. *)
DI_VALVE_CLOSED          AT %I2.0 : BOOL;
DI_PUMP_STOPPED          AT %I2.1 : BOOL;
DI_BYPASS_SAFE_FB        AT %I2.2 : BOOL;
DI_GENERATOR_SAFE_FB     AT %I2.3 : BOOL;
DI_VALVE_TRACKING        AT %I2.4 : BOOL;
DI_PUMP_TRACKING         AT %I2.5 : BOOL;
DI_LOAD1_TRACKING        AT %I2.6 : BOOL;
DI_LOAD2_TRACKING        AT %I2.7 : BOOL;

(* Physical digital outputs. *)
DO_PUMP_ENABLE           AT %Q0.0 : BOOL;
DO_LOAD_ENABLE           AT %Q0.1 : BOOL;
DO_BYPASS_SAFE           AT %Q0.2 : BOOL;
DO_TRIP_RELAY            AT %Q0.3 : BOOL;
DO_ALARM                 AT %Q0.4 : BOOL;
DO_WATCHDOG              AT %Q0.5 : BOOL;

(*
   Module/device quality flags.
   Map these from the real I/O diagnostics or communications stack.
*)
Q_P1, Q_P2A, Q_P2B, Q_P2C, Q_PA : BOOL;
Q_T1, Q_T2, Q_TA : BOOL;
Q_RH1, Q_RH2, Q_RHA : BOOL;
Q_U1, Q_LIQFLOW, Q_VALVE_POS : BOOL;
Q_PEL1, Q_PEL2, Q_PPUMP, Q_PAUX : BOOL;
Q_VIB1, Q_VIB2, Q_CARRY : BOOL;
Q_HSC_N1, Q_HSC_N2 : BOOL;

(* Stale-data flags supplied by the device or communications layer. *)
```

```
    STALE_P1, STALE_P2A, STALE_P2B, STALE_P2C, STALE_PA : BOOL;
    STALE_T1, STALE_T2, STALE_TA : BOOL;
    STALE_RH1, STALE_RH2, STALE_RHA : BOOL;
    STALE_U1, STALE_LIQFLOW, STALE_VALVE_POS : BOOL;
    STALE_PEL1, STALE_PEL2, STALE_PPUMP, STALE_PAUX : BOOL;
    STALE_VIB1, STALE_VIB2, STALE_CARRY : BOOL;
    STALE_HSC_N1, STALE_HSC_N2 : BOOL;

    (*
       Actual elapsed task time supplied by the PLC runtime.
       The integration period is not assumed to be exactly 50 ms.

       SYS_TASK_DT_MS must be explicitly populated from the selected
       PLC runtime clock or cyclic-task diagnostic. It is not an
       automatically portable IEC 61131-3 system variable.
    *)
    SYS_TASK_DT_MS : DINT;
END_VAR


FUNCTION ClampReal : REAL
VAR_INPUT
    X  : REAL;
    Lo : REAL;
    Hi : REAL;
END_VAR

IF X < Lo THEN
    ClampReal := Lo;
ELSIF X > Hi THEN
    ClampReal := Hi;
ELSE
    ClampReal := X;
END_IF;
END_FUNCTION


FUNCTION RawAIInTolerance : BOOL
VAR_INPUT
    Raw : INT;
END_VAR

(*
   The normal configured 4–20 mA span is 0–27,648.
   The wider values below provide limited numerical tolerance only.
   Module quality and under-range/over-range diagnostics remain primary.
*)
RawAIInTolerance :=
    (Raw >= -500)
    AND
    (Raw <= 28150);
END_FUNCTION


FUNCTION Scale4_20mA : REAL
VAR_INPUT
    Raw   : INT;
    EngLo : REAL;
    EngHi : REAL;
END_VAR
VAR
    R : REAL;
END_VAR

R := INT_TO_REAL(Raw);

Scale4_20mA :=
    EngLo
    +
    (R / 27648.0)
    *
    (EngHi - EngLo);
END_FUNCTION


FUNCTION ToAO4_20mA : INT
VAR_INPUT
    Percent : REAL;
```

```
END_VAR
VAR
    P : REAL;
END_VAR

P := ClampReal(
    X := Percent,
    Lo := 0.0,
    Hi := 100.0);

ToAO4_20mA :=
    REAL_TO_INT(
        (P / 100.0)
        *
        27648.0);
END_FUNCTION


FUNCTION Median3 : REAL
VAR_INPUT
    A : REAL;
    B : REAL;
    C : REAL;
END_VAR

IF A > B THEN
    IF B > C THEN
        Median3 := B;
    ELSIF A > C THEN
        Median3 := C;
    ELSE
        Median3 := A;
    END_IF;
ELSE
    IF A > C THEN
        Median3 := A;
    ELSIF B > C THEN
        Median3 := C;
    ELSE
        Median3 := B;
    END_IF;
END_IF;
END_FUNCTION


FUNCTION RateLimit : REAL
VAR_INPUT
    Target    : REAL;
    Previous  : REAL;
    RisePerS  : REAL;
    FallPerS  : REAL;
    Ts        : REAL;
END_VAR
VAR
    Delta : REAL;
END_VAR

Delta := Target - Previous;

IF Delta > RisePerS * Ts THEN
    RateLimit := Previous + RisePerS * Ts;
ELSIF Delta < -FallPerS * Ts THEN
    RateLimit := Previous - FallPerS * Ts;
ELSE
    RateLimit := Target;
END_IF;
END_FUNCTION


FUNCTION SaturationPressureKPa : REAL
VAR_INPUT
    TemperatureC : REAL;
END_VAR
VAR
    Tuse : REAL;
END_VAR

(*
```

```
   Preliminary air–water correlation for approximately -40 to 100 °C.
   Replace with a validated fluid-property library where the gas,
   liquid, temperature or pressure falls outside that range.
*)
Tuse := ClampReal(
    X := TemperatureC,
    Lo := -40.0,
    Hi := 100.0);

SaturationPressureKPa :=
    0.61121
    *
    EXP(
        (18.678 - Tuse / 234.5)
        *
        (Tuse / (257.14 + Tuse)));
END_FUNCTION


FUNCTION HumidityRatio : REAL
VAR_INPUT
    RelativeHumidityPct : REAL;
    TemperatureC        : REAL;
    PressureKPa         : REAL;
END_VAR
VAR
    RHfrac : REAL;
    Psat   : REAL;
    Pv     : REAL;
END_VAR

RHfrac :=
    ClampReal(
        X := RelativeHumidityPct / 100.0,
        Lo := 0.0,
        Hi := 1.0);

Psat := SaturationPressureKPa(TemperatureC := TemperatureC);
Pv := RHfrac * Psat;

IF PressureKPa <= Pv + 0.01 THEN
    (* Negative value is an invalid-state sentinel, not a physical ratio. *)
    HumidityRatio := -1.0;
ELSE
    HumidityRatio :=
        0.621945
        *
        Pv
        /
        (PressureKPa - Pv);
END_IF;
END_FUNCTION


FUNCTION MoistAirDensity : REAL
VAR_INPUT
    PressureKPa  : REAL;
    TemperatureC : REAL;
    HumRatio     : REAL;
END_VAR
VAR
    TemperatureK : REAL;
    Rmix         : REAL;
END_VAR

TemperatureK := TemperatureC + 273.15;

Rmix :=
    (287.055 + HumRatio * 461.495)
    /
    (1.0 + HumRatio);

IF TemperatureK <= 0.0 THEN
    MoistAirDensity := 0.0;
ELSE
    MoistAirDensity :=
        PressureKPa * 1000.0
        /
```

```
        (Rmix * TemperatureK);
END_IF;
END_FUNCTION


FUNCTION_BLOCK FB_ValidateSignal
VAR_INPUT
    Value       : REAL;
    EngineeringLo : REAL;
    EngineeringHi : REAL;
    MaximumRate : REAL;
    Ts          : REAL;
    QualityOK   : BOOL;
    Stale       : BOOL;
    FirstScan   : BOOL;
END_VAR

VAR_OUTPUT
    Valid   : BOOL;
    RangeOK : BOOL;
    RateOK  : BOOL;
END_VAR

VAR
    PreviousValue : REAL;
END_VAR

IF FirstScan THEN
    PreviousValue := Value;
END_IF;

RangeOK :=
    (Value >= EngineeringLo)
    AND
    (Value <= EngineeringHi);

RateOK :=
    FirstScan
    OR
    (
        ABS(Value - PreviousValue)
        <=
        MaximumRate * Ts
    );

Valid :=
    QualityOK
    AND
    NOT Stale
    AND
    RangeOK
    AND
    RateOK;

PreviousValue := Value;
END_FUNCTION_BLOCK


FUNCTION_BLOCK FB_P2Vote
VAR_INPUT
    A, B, C            : REAL;
    ValidA, ValidB,
    ValidC             : BOOL;
    MaximumSpreadKPa   : REAL;
    LowTripLimitKPa    : REAL;
END_VAR

VAR_OUTPUT
    ControlPV       : REAL;
    SpreadKPa       : REAL;
    ValidCount      : INT;
    VoteOK          : BOOL;
    Degraded        : BOOL;
    LowTrip2oo3     : BOOL;
    OutlierA        : BOOL;
    OutlierB        : BOOL;
    OutlierC        : BOOL;
END_VAR
```

```
VAR
    DAB, DAC, DBC : REAL;
    ClosestPairDifference : REAL;
END_VAR

ValidCount := 0;
VoteOK := FALSE;
Degraded := TRUE;
OutlierA := FALSE;
OutlierB := FALSE;
OutlierC := FALSE;

IF ValidA THEN
    ValidCount := ValidCount + 1;
END_IF;

IF ValidB THEN
    ValidCount := ValidCount + 1;
END_IF;

IF ValidC THEN
    ValidCount := ValidCount + 1;
END_IF;

(*
   Independent 2-out-of-3 low-pressure decision.
   A trip requires two valid channels at or below the trip threshold.
*)
LowTrip2oo3 :=
    (
        ValidA
        AND ValidB
        AND (A <= LowTripLimitKPa)
        AND (B <= LowTripLimitKPa)
    )
    OR
    (
        ValidA
        AND ValidC
        AND (A <= LowTripLimitKPa)
        AND (C <= LowTripLimitKPa)
    )
    OR
    (
        ValidB
        AND ValidC
        AND (B <= LowTripLimitKPa)
        AND (C <= LowTripLimitKPa)
    );

IF ValidCount = 3 THEN
    DAB := ABS(A - B);
    DAC := ABS(A - C);
    DBC := ABS(B - C);

    SpreadKPa :=
        MAX(A, MAX(B, C))
        -
        MIN(A, MIN(B, C));

    IF SpreadKPa <= MaximumSpreadKPa THEN
        (* All three channels agree. *)
        ControlPV := Median3(A := A, B := B, C := C);
        VoteOK := TRUE;
        Degraded := FALSE;

    ELSE
        (* Select the closest agreeing pair and identify one outlier. *)
        ClosestPairDifference := MIN(DAB, MIN(DAC, DBC));

        IF ClosestPairDifference <= MaximumSpreadKPa THEN
            VoteOK := TRUE;
            Degraded := TRUE;

            IF (DAB <= DAC) AND (DAB <= DBC) THEN
                ControlPV := 0.5 * (A + B);
                OutlierC := TRUE;
```

```
            ELSIF (DAC <= DAB) AND (DAC <= DBC) THEN
                ControlPV := 0.5 * (A + C);
                OutlierB := TRUE;

            ELSE
                ControlPV := 0.5 * (B + C);
                OutlierA := TRUE;
            END_IF;

        ELSE
            (* No two channels agree within the commissioned tolerance. *)
            ControlPV := Median3(A := A, B := B, C := C);
            VoteOK := FALSE;
            Degraded := TRUE;
        END_IF;
    END_IF;

ELSIF ValidCount = 2 THEN
    Degraded := TRUE;

    IF ValidA AND ValidB THEN
        ControlPV := 0.5 * (A + B);
        SpreadKPa := ABS(A - B);
        OutlierC := TRUE;

    ELSIF ValidA AND ValidC THEN
        ControlPV := 0.5 * (A + C);
        SpreadKPa := ABS(A - C);
        OutlierB := TRUE;

    ELSE
        ControlPV := 0.5 * (B + C);
        SpreadKPa := ABS(B - C);
        OutlierA := TRUE;
    END_IF;

    VoteOK := SpreadKPa <= MaximumSpreadKPa;

ELSE
    ControlPV := 0.0;
    SpreadKPa := 9999.0;
    VoteOK := FALSE;
    Degraded := TRUE;
END_IF;
END_FUNCTION_BLOCK


FUNCTION_BLOCK FB_PID_AW_DOnPV
VAR_INPUT
    Enable        : BOOL;
    Setpoint      : REAL;
    ProcessValue  : REAL;
    Kp            : REAL;
    Ki            : REAL;
    Kd            : REAL;
    Kaw           : REAL;
    DerivativeTau : REAL;
    Ts            : REAL;
    OutputLo      : REAL;
    OutputHi      : REAL;
END_VAR

VAR_OUTPUT
    Output : REAL;
    Error  : REAL;
END_VAR

VAR
    Integral      : REAL;
    PreviousPV    : REAL;
    DerivativePV  : REAL;
    DerivativeRaw : REAL;
    AlphaD        : REAL;
    RawOutput     : REAL;
    WasEnabled    : BOOL;
END_VAR
```

```
(*
   Error sign:
       Error = P2 - P2,set

   Positive pressure error permits an increase in spray.
   The process gain is assumed negative because increased spray
   reduces Zone 2 pressure.
*)

IF NOT Enable THEN
    Integral := 0.0;
    PreviousPV := ProcessValue;
    DerivativePV := 0.0;
    Output := 0.0;
    Error := 0.0;
    WasEnabled := FALSE;

ELSE
    Error := ProcessValue - Setpoint;

    IF NOT WasEnabled THEN
        PreviousPV := ProcessValue;
        DerivativePV := 0.0;
        WasEnabled := TRUE;
    END_IF;

    IF Ts > 0.0 THEN
        DerivativeRaw :=
            (ProcessValue - PreviousPV)
            /
            Ts;
    ELSE
        DerivativeRaw := 0.0;
    END_IF;

    AlphaD :=
        Ts
        /
        MAX(DerivativeTau + Ts, 0.001);

    DerivativePV :=
        DerivativePV
        +
        AlphaD
        *
        (DerivativeRaw - DerivativePV);

    RawOutput :=
        Kp * Error
        +
        Integral
        +
        Kd * DerivativePV;

    Output :=
        ClampReal(
            X := RawOutput,
            Lo := OutputLo,
            Hi := OutputHi);

    Integral :=
        Integral
        +
        Ts
        *
        (
            Ki * Error
            +
            Kaw * (Output - RawOutput)
        );

    PreviousPV := ProcessValue;
END_IF;
END_FUNCTION_BLOCK


PROGRAM PRG_EnergyRegulator
```

```
VAR CONSTANT
    TS_NOMINAL_S              : REAL := 0.05;
    MAX_TASK_DT_S             : REAL := 0.20;

    PRESSURE_FILTER_TAU_S     : REAL := 0.25;
    POWER_FILTER_TAU_S        : REAL := 0.50;

    FP_NORMAL                 : REAL := 1.030;
    FTRIP                     : REAL := 1.010;

    P2_SPREAD_MAX_KPA         : REAL := 1.00;
    PRESSURE_CONTROL_BAND_KPA : REAL := 0.50;
    PRESSURE_RESERVE_KPA      : REAL := 1.00;
    RESET_PRESSURE_MARGIN_KPA : REAL := 0.50;

    N1_TRIP_RPM               : REAL := 18000.0;
    N2_TRIP_RPM               : REAL := 18000.0;
    N1_RESERVE_RPM            : REAL := 900.0;
    N2_RESERVE_RPM            : REAL := 900.0;
    SPEED_SETTLED_RATE_RPM_S  : REAL := 100.0;

    T1_MAX_C                  : REAL := 250.0;
    T2_MIN_C                  : REAL := -20.0;

    RH2_MAX_PCT               : REAL := 98.0;
    VIBRATION_MAX_MM_S        : REAL := 20.0;
    CARRYOVER_MAX_KG_S        : REAL := 0.01;

    INLET_AREA_M2             : REAL := 1.0;
    N1_PULSES_PER_REV         : REAL := 60.0;
    N2_PULSES_PER_REV         : REAL := 60.0;

    KP_PRESSURE               : REAL := 2.0;
    KI_PRESSURE               : REAL := 0.25;
    KD_PRESSURE               : REAL := 0.05;
    KAW_PRESSURE              : REAL := 1.0;
    DERIVATIVE_TAU_S          : REAL := 0.20;

    KFF_DELTA_T               : REAL := 0.15;
    KFF_INLET_VELOCITY        : REAL := 0.20;

    LOAD_BASE1_PCT            : REAL := 50.0;
    LOAD_BASE2_PCT            : REAL := 50.0;
    LOAD_SPEED_GAIN1          : REAL := 0.015;
    LOAD_SPEED_GAIN2          : REAL := 0.015;
    LOAD_REFERENCE1_RPM       : REAL := 12000.0;
    LOAD_REFERENCE2_RPM       : REAL := 12000.0;

    POWER_IMPROVEMENT_MIN_KW  : REAL := 0.50;
    OPT_MIN_STEP_PCT          : REAL := 0.10;
    OPT_MAX_BIAS_PCT          : REAL := 20.0;

    RESTART_DWELL_TIME        : TIME := T#10s;
    SAFE_PROOF_TIME           : TIME := T#3s;
    WATCHDOG_HALF_PERIOD      : TIME := T#500ms;

    OPT_BASELINE_TIME         : TIME := T#2s;
    OPT_ACTUATOR_TIME         : TIME := T#2s;
    OPT_SETTLE_TIME           : TIME := T#3s;
    OPT_SAMPLE_TIME           : TIME := T#2s;
    OPT_TIMEOUT_TIME          : TIME := T#30s;

    (*
       Cause-specific safe generator commands.

       The zero defaults are de-energised placeholders only.
       They are not a recommendation for an operating turbine.
       Final values and load-enable states must be established by:
       - overspeed studies;
       - loss-of-grid studies;
       - converter studies;
       - braking-resistor or dump-load studies; and
       - validated bypass and shutdown analysis.
    *)
    SAFE_PROCESS_LOAD1_PCT    : REAL := 0.0;
    SAFE_PROCESS_LOAD2_PCT    : REAL := 0.0;
    SAFE_PROCESS_LOAD_ENABLE  : BOOL := FALSE;
```

```
    SAFE_OVERSPEED_LOAD1_PCT  : REAL := 0.0;
    SAFE_OVERSPEED_LOAD2_PCT  : REAL := 0.0;
    SAFE_OVERSPEED_LOAD_ENABLE: BOOL := FALSE;
END_VAR


VAR
    (* Task timing. *)
    ActualElapsedTimeS : REAL;
    ControlTsS         : REAL;
    TaskTimeOK         : BOOL;
    FirstScan          : BOOL := TRUE;

    (* Scaled engineering values. *)
    P1, P2A, P2B, P2C, Pa : REAL;
    T1, T2, Ta             : REAL;
    RH1, RH2, RHa          : REAL;
    U1                     : REAL;
    LiquidFlowKgS          : REAL;
    ValvePositionPct       : REAL;

    Pel1, Pel2             : REAL;
    Ppump, Paux            : REAL;
    Vibration1, Vibration2 : REAL;
    CarryoverKgS           : REAL;

    N1, N2                 : REAL;
    N1FrequencyHz          : REAL;
    N2FrequencyHz          : REAL;
    N1RateRPMs             : REAL;
    N2RateRPMs             : REAL;
    PreviousN1             : REAL;
    PreviousN2             : REAL;

    (* Psychrometric and flow calculations. *)
    PropertyModelOK        : BOOL;
    HumidityRatio1         : REAL;
    HumidityRatio2         : REAL;
    HumidityRatioAmbient   : REAL;
    HumidityRatioSat2      : REAL;
    SaturationMargin       : REAL;
    Density1               : REAL;
    TotalMassFlowKgS       : REAL;
    DryAirMassFlowKgS      : REAL;
    EvapCapacityKgS        : REAL;
    LiquidCapacityRatio    : REAL;

    (* Signal-validation blocks. *)
    SV_P1, SV_P2A, SV_P2B, SV_P2C, SV_PA : FB_ValidateSignal;
    SV_T1, SV_T2, SV_TA : FB_ValidateSignal;
    SV_RH1, SV_RH2, SV_RHA : FB_ValidateSignal;
    SV_U1, SV_LIQFLOW, SV_VALVEPOS : FB_ValidateSignal;
    SV_PEL1, SV_PEL2, SV_PPUMP, SV_PAUX : FB_ValidateSignal;
    SV_VIB1, SV_VIB2, SV_CARRY : FB_ValidateSignal;
    SV_N1, SV_N2 : FB_ValidateSignal;

    (* Zone 2 pressure voting and filtering. *)
    P2Vote                 : FB_P2Vote;
    PressurePID            : FB_PID_AW_DOnPV;

    P2Set                  : REAL;
    P2Trip                 : REAL;
    P2ControlRaw           : REAL;
    P2ControlFilt          : REAL;
    P2SpreadKPa            : REAL;
    PressureFilterAlpha    : REAL;
    PowerFilterAlpha       : REAL;
    PressureVoteOK         : BOOL;
    PressureVoteDegraded   : BOOL;
    P2LowTrip2oo3          : BOOL;
    P2OutlierA             : BOOL;
    P2OutlierB             : BOOL;
    P2OutlierC             : BOOL;

    (* Power and energy. *)
    Pnet                   : REAL;
    PnetFilt               : REAL;
    EnergyKWh              : LREAL;
```

```
PowerMeasurementsOK    : BOOL;

(* Health, constraints and protective conditions. *)
CriticalSensorsOK      : BOOL;
OptimisationSensorsOK  : BOOL;
PlantReady             : BOOL;

PressureReserveLow     : BOOL;
Speed1ReserveLow       : BOOL;
Speed2ReserveLow       : BOOL;
TemperatureReserveLow  : BOOL;
VibrationReserveLow    : BOOL;
CarryoverReserveLow    : BOOL;
SaturationReserveLow   : BOOL;
TurbineMapReserveLow   : BOOL;
ElectricalReserveLow   : BOOL;

NearConstraint         : BOOL;
ProcessSettled         : BOOL;
AllActuatorsTracking   : BOOL;

TemperatureTrip        : BOOL;
VibrationTrip          : BOOL;
CarryoverTrip          : BOOL;
DriveFault             : BOOL;
CriticalSensorFault    : BOOL;
WatchdogFault          : BOOL;
FinalElementFault      : BOOL;
FinalElementFaultLatched : BOOL := FALSE;

TripRequest            : BOOL;
TripLatched            : BOOL := TRUE;
CurrentTripCause       : E_TripCause := TC_NONE;
ActiveTripCause        : E_TripCause := TC_NONE;

(* Restart and final-element proof. *)
ResetConditionsOK      : BOOL;
SafeStateProven        : BOOL;
ResetDwellTimer        : TON;
SafeProofTimer         : TON;

(* Watchdog. *)
WatchdogTimer          : TON;
WatchdogReset          : BOOL := FALSE;
WatchdogBit            : BOOL := FALSE;

(* Normal control commands. *)
ValveFFPct             : REAL;
ValvePIDPct            : REAL;
ValveBiasPct           : REAL := 0.0;
ValveTargetPct         : REAL;
ValveCmdPct            : REAL;

PumpTargetPct          : REAL;
PumpCmdPct             : REAL;

Load1BiasPct           : REAL := 0.0;
Load2BiasPct           : REAL := 0.0;
Load1TargetPct         : REAL;
Load2TargetPct         : REAL;
Load1CmdPct            : REAL;
Load2CmdPct            : REAL;

(* Cause-specific safe commands. *)
SafeLoad1Pct           : REAL;
SafeLoad2Pct           : REAL;
SafeLoadEnable         : BOOL;

(* Optimiser. *)
OptState               : E_OptimizerState := OPT_BASELINE;
OptEnabled             : BOOL;
OptIndex               : INT := 0;

OptDirection           : ARRAY[0..2] OF REAL :=
                         [1.0, 1.0, 1.0];

TrialStepPct           : REAL := 0.50;
OldBiasPct             : REAL := 0.0;
```

```
    BaselinePowerSum       : LREAL := 0.0;
    TrialPowerSum          : LREAL := 0.0;
    BaselineSampleCount    : DINT := 0;
    TrialSampleCount       : DINT := 0;
    BaselinePowerAvg       : REAL := 0.0;
    TrialPowerAvg          : REAL := 0.0;

    BaselineTimer          : TON;
    ActuatorTimer          : TON;
    SettlingTimer          : TON;
    TrialSampleTimer       : TON;
    OptimiserTimeoutTimer  : TON;

    (* General alarms. *)
    DegradedOperation      : BOOL;
END_VAR

(*********************************************************************
 1. ACTUAL TASK TIME
*********************************************************************)

ActualElapsedTimeS :=
    DINT_TO_REAL(SYS_TASK_DT_MS)
    /
    1000.0;

TaskTimeOK :=
    (ActualElapsedTimeS > 0.001)
    AND
    (ActualElapsedTimeS <= MAX_TASK_DT_S);

IF TaskTimeOK THEN
    ControlTsS := ActualElapsedTimeS;
ELSE
    ControlTsS := TS_NOMINAL_S;
END_IF;


(*********************************************************************
 2. SCALE PHYSICAL INPUTS
*********************************************************************)

P1  := Scale4_20mA(AI_P1_RAW,  80.0, 300.0);
P2A := Scale4_20mA(AI_P2A_RAW, 80.0, 200.0);
P2B := Scale4_20mA(AI_P2B_RAW, 80.0, 200.0);
P2C := Scale4_20mA(AI_P2C_RAW, 80.0, 200.0);
Pa  := Scale4_20mA(AI_PA_RAW,  80.0, 120.0);

T1 := Scale4_20mA(AI_T1_RAW, -40.0, 250.0);
T2 := Scale4_20mA(AI_T2_RAW, -40.0, 250.0);
Ta := Scale4_20mA(AI_TA_RAW, -40.0,  80.0);

RH1 := Scale4_20mA(AI_RH1_RAW, 0.0, 100.0);
RH2 := Scale4_20mA(AI_RH2_RAW, 0.0, 100.0);
RHa := Scale4_20mA(AI_RHA_RAW, 0.0, 100.0);

U1 :=
    Scale4_20mA(
        AI_U1_RAW,
        0.0,
        50.0);

LiquidFlowKgS :=
    Scale4_20mA(
        AI_LIQFLOW_RAW,
        0.0,
        1.0);

ValvePositionPct :=
    Scale4_20mA(
        AI_VALVE_POS_RAW,
        0.0,
        100.0);

Pel1 :=
    Scale4_20mA(
```

```
        AI_PEL1_RAW,
        0.0,
        500.0);

Pel2 :=
    Scale4_20mA(
        AI_PEL2_RAW,
        0.0,
        500.0);

Ppump :=
    Scale4_20mA(
        AI_PPUMP_RAW,
        0.0,
        100.0);

Paux :=
    Scale4_20mA(
        AI_PAUX_RAW,
        0.0,
        100.0);

Vibration1 :=
    Scale4_20mA(
        AI_VIB1_RAW,
        0.0,
        50.0);

Vibration2 :=
    Scale4_20mA(
        AI_VIB2_RAW,
        0.0,
        50.0);

CarryoverKgS :=
    Scale4_20mA(
        AI_CARRY_RAW,
        0.0,
        0.10);


(************************************************************************
 3. HIGH-SPEED-COUNTER TURBINE SPEEDS
************************************************************************)

N1FrequencyHz :=
    DINT_TO_REAL(HSC_N1_FREQ_MHZ)
    /
    1000.0;

N2FrequencyHz :=
    DINT_TO_REAL(HSC_N2_FREQ_MHZ)
    /
    1000.0;

IF N1_PULSES_PER_REV > 0.0 THEN
    N1 :=
        N1FrequencyHz
        *
        60.0
        /
        N1_PULSES_PER_REV;
ELSE
    N1 := 0.0;
END_IF;

IF N2_PULSES_PER_REV > 0.0 THEN
    N2 :=
        N2FrequencyHz
        *
        60.0
        /
        N2_PULSES_PER_REV;
ELSE
    N2 := 0.0;
END_IF;

IF FirstScan OR ControlTsS <= 0.0 THEN
```

```
    PreviousN1 := N1;
    PreviousN2 := N2;
    N1RateRPMs := 0.0;
    N2RateRPMs := 0.0;
ELSE
    N1RateRPMs :=
        (N1 - PreviousN1)
        /
        ControlTsS;

    N2RateRPMs :=
        (N2 - PreviousN2)
        /
        ControlTsS;
END_IF;


(************************************************************************
 4. INDIVIDUAL SIGNAL VALIDATION
************************************************************************)

SV_P1(
    Value := P1,
    EngineeringLo := 80.0,
    EngineeringHi := 300.0,
    MaximumRate := 100.0,
    Ts := ControlTsS,
    QualityOK := Q_P1 AND RawAIInTolerance(AI_P1_RAW),
    Stale := STALE_P1,
    FirstScan := FirstScan);

SV_P2A(
    Value := P2A,
    EngineeringLo := 80.0,
    EngineeringHi := 200.0,
    MaximumRate := 100.0,
    Ts := ControlTsS,
    QualityOK := Q_P2A AND RawAIInTolerance(AI_P2A_RAW),
    Stale := STALE_P2A,
    FirstScan := FirstScan);

SV_P2B(
    Value := P2B,
    EngineeringLo := 80.0,
    EngineeringHi := 200.0,
    MaximumRate := 100.0,
    Ts := ControlTsS,
    QualityOK := Q_P2B AND RawAIInTolerance(AI_P2B_RAW),
    Stale := STALE_P2B,
    FirstScan := FirstScan);

SV_P2C(
    Value := P2C,
    EngineeringLo := 80.0,
    EngineeringHi := 200.0,
    MaximumRate := 100.0,
    Ts := ControlTsS,
    QualityOK := Q_P2C AND RawAIInTolerance(AI_P2C_RAW),
    Stale := STALE_P2C,
    FirstScan := FirstScan);

SV_PA(
    Value := Pa,
    EngineeringLo := 80.0,
    EngineeringHi := 120.0,
    MaximumRate := 5.0,
    Ts := ControlTsS,
    QualityOK := Q_PA AND RawAIInTolerance(AI_PA_RAW),
    Stale := STALE_PA,
    FirstScan := FirstScan);

SV_T1(
    Value := T1,
    EngineeringLo := -40.0,
    EngineeringHi := 250.0,
    MaximumRate := 100.0,
    Ts := ControlTsS,
    QualityOK := Q_T1 AND RawAIInTolerance(AI_T1_RAW),
```

```
    Stale := STALE_T1,
    FirstScan := FirstScan);

SV_T2(
    Value := T2,
    EngineeringLo := -40.0,
    EngineeringHi := 250.0,
    MaximumRate := 100.0,
    Ts := ControlTsS,
    QualityOK := Q_T2 AND RawAIInTolerance(AI_T2_RAW),
    Stale := STALE_T2,
    FirstScan := FirstScan);

SV_TA(
    Value := Ta,
    EngineeringLo := -40.0,
    EngineeringHi := 80.0,
    MaximumRate := 20.0,
    Ts := ControlTsS,
    QualityOK := Q_TA AND RawAIInTolerance(AI_TA_RAW),
    Stale := STALE_TA,
    FirstScan := FirstScan);

SV_RH1(
    Value := RH1,
    EngineeringLo := 0.0,
    EngineeringHi := 100.0,
    MaximumRate := 100.0,
    Ts := ControlTsS,
    QualityOK := Q_RH1 AND RawAIInTolerance(AI_RH1_RAW),
    Stale := STALE_RH1,
    FirstScan := FirstScan);

SV_RH2(
    Value := RH2,
    EngineeringLo := 0.0,
    EngineeringHi := 100.0,
    MaximumRate := 100.0,
    Ts := ControlTsS,
    QualityOK := Q_RH2 AND RawAIInTolerance(AI_RH2_RAW),
    Stale := STALE_RH2,
    FirstScan := FirstScan);

SV_RHA(
    Value := RHa,
    EngineeringLo := 0.0,
    EngineeringHi := 100.0,
    MaximumRate := 100.0,
    Ts := ControlTsS,
    QualityOK := Q_RHA AND RawAIInTolerance(AI_RHA_RAW),
    Stale := STALE_RHA,
    FirstScan := FirstScan);

SV_U1(
    Value := U1,
    EngineeringLo := 0.0,
    EngineeringHi := 50.0,
    MaximumRate := 100.0,
    Ts := ControlTsS,
    QualityOK := Q_U1 AND RawAIInTolerance(AI_U1_RAW),
    Stale := STALE_U1,
    FirstScan := FirstScan);

SV_LIQFLOW(
    Value := LiquidFlowKgS,
    EngineeringLo := 0.0,
    EngineeringHi := 1.0,
    MaximumRate := 2.0,
    Ts := ControlTsS,
    QualityOK :=
        Q_LIQFLOW
        AND RawAIInTolerance(AI_LIQFLOW_RAW),
    Stale := STALE_LIQFLOW,
    FirstScan := FirstScan);

SV_VALVEPOS(
    Value := ValvePositionPct,
    EngineeringLo := 0.0,
```

```
    EngineeringHi := 100.0,
    MaximumRate := 200.0,
    Ts := ControlTsS,
    QualityOK :=
        Q_VALVE_POS
        AND RawAIInTolerance(AI_VALVE_POS_RAW),
    Stale := STALE_VALVE_POS,
    FirstScan := FirstScan);

SV_PEL1(
    Value := Pel1,
    EngineeringLo := 0.0,
    EngineeringHi := 500.0,
    MaximumRate := 1000.0,
    Ts := ControlTsS,
    QualityOK := Q_PEL1 AND RawAIInTolerance(AI_PEL1_RAW),
    Stale := STALE_PEL1,
    FirstScan := FirstScan);

SV_PEL2(
    Value := Pel2,
    EngineeringLo := 0.0,
    EngineeringHi := 500.0,
    MaximumRate := 1000.0,
    Ts := ControlTsS,
    QualityOK := Q_PEL2 AND RawAIInTolerance(AI_PEL2_RAW),
    Stale := STALE_PEL2,
    FirstScan := FirstScan);

SV_PPUMP(
    Value := Ppump,
    EngineeringLo := 0.0,
    EngineeringHi := 100.0,
    MaximumRate := 500.0,
    Ts := ControlTsS,
    QualityOK := Q_PPUMP AND RawAIInTolerance(AI_PPUMP_RAW),
    Stale := STALE_PPUMP,
    FirstScan := FirstScan);

SV_PAUX(
    Value := Paux,
    EngineeringLo := 0.0,
    EngineeringHi := 100.0,
    MaximumRate := 500.0,
    Ts := ControlTsS,
    QualityOK := Q_PAUX AND RawAIInTolerance(AI_PAUX_RAW),
    Stale := STALE_PAUX,
    FirstScan := FirstScan);

SV_VIB1(
    Value := Vibration1,
    EngineeringLo := 0.0,
    EngineeringHi := 50.0,
    MaximumRate := 200.0,
    Ts := ControlTsS,
    QualityOK := Q_VIB1 AND RawAIInTolerance(AI_VIB1_RAW),
    Stale := STALE_VIB1,
    FirstScan := FirstScan);

SV_VIB2(
    Value := Vibration2,
    EngineeringLo := 0.0,
    EngineeringHi := 50.0,
    MaximumRate := 200.0,
    Ts := ControlTsS,
    QualityOK := Q_VIB2 AND RawAIInTolerance(AI_VIB2_RAW),
    Stale := STALE_VIB2,
    FirstScan := FirstScan);

SV_CARRY(
    Value := CarryoverKgS,
    EngineeringLo := 0.0,
    EngineeringHi := 0.10,
    MaximumRate := 1.0,
    Ts := ControlTsS,
    QualityOK := Q_CARRY AND RawAIInTolerance(AI_CARRY_RAW),
    Stale := STALE_CARRY,
    FirstScan := FirstScan);
```

```
SV_N1(
    Value := N1,
    EngineeringLo := 0.0,
    EngineeringHi := 20000.0,
    MaximumRate := 100000.0,
    Ts := ControlTsS,
    QualityOK := Q_HSC_N1,
    Stale := STALE_HSC_N1,
    FirstScan := FirstScan);

SV_N2(
    Value := N2,
    EngineeringLo := 0.0,
    EngineeringHi := 20000.0,
    MaximumRate := 100000.0,
    Ts := ControlTsS,
    QualityOK := Q_HSC_N2,
    Stale := STALE_HSC_N2,
    FirstScan := FirstScan);


(************************************************************************
 5. PRESSURE SETPOINTS AND VALID-CHANNEL VOTING
************************************************************************)

P2Set := FP_NORMAL * Pa;
P2Trip := FTRIP * Pa;

P2Vote(
    A := P2A,
    B := P2B,
    C := P2C,
    ValidA := SV_P2A.Valid,
    ValidB := SV_P2B.Valid,
    ValidC := SV_P2C.Valid,
    MaximumSpreadKPa := P2_SPREAD_MAX_KPA,
    LowTripLimitKPa := P2Trip);

P2ControlRaw := P2Vote.ControlPV;
P2SpreadKPa := P2Vote.SpreadKPa;
PressureVoteOK := P2Vote.VoteOK;
PressureVoteDegraded := P2Vote.Degraded;
P2LowTrip2oo3 := P2Vote.LowTrip2oo3;
P2OutlierA := P2Vote.OutlierA;
P2OutlierB := P2Vote.OutlierB;
P2OutlierC := P2Vote.OutlierC;


(************************************************************************
 6. FILTERED CONTROL PATH AND UNFILTERED PROTECTIVE PATH
************************************************************************)

PressureFilterAlpha :=
    ControlTsS
    /
    MAX(
        PRESSURE_FILTER_TAU_S + ControlTsS,
        0.001);

PowerFilterAlpha :=
    ControlTsS
    /
    MAX(
        POWER_FILTER_TAU_S + ControlTsS,
        0.001);

Pnet := Pel1 + Pel2 - Ppump - Paux;

IF FirstScan THEN
    P2ControlFilt := P2ControlRaw;
    PnetFilt := Pnet;
ELSE
    P2ControlFilt :=
        P2ControlFilt
        +
        PressureFilterAlpha
        *
```

```
        (P2ControlRaw - P2ControlFilt);

    PnetFilt :=
        PnetFilt
        +
        PowerFilterAlpha
        *
        (Pnet - PnetFilt);
END_IF;

PowerMeasurementsOK :=
    SV_PEL1.Valid
    AND SV_PEL2.Valid
    AND SV_PPUMP.Valid
    AND SV_PAUX.Valid;

IF PowerMeasurementsOK AND TaskTimeOK THEN
    EnergyKWh :=
        EnergyKWh
        +
        REAL_TO_LREAL(Pnet)
        *
        REAL_TO_LREAL(ActualElapsedTimeS)
        /
        3600.0;
END_IF;


(**********************************************************************
 7. HUMIDITY, DENSITY AND EVAPORATION-CAPACITY ESTIMATES
**********************************************************************)

PropertyModelOK :=
    PressureVoteOK
    AND SV_P1.Valid
    AND SV_PA.Valid
    AND SV_T1.Valid
    AND SV_T2.Valid
    AND SV_TA.Valid
    AND SV_RH1.Valid
    AND SV_RH2.Valid
    AND SV_RHA.Valid
    AND (P2ControlRaw > 0.0)
    AND (T1 >= -40.0)
    AND (T1 <= 100.0)
    AND (T2 >= -40.0)
    AND (T2 <= 100.0)
    AND (Ta >= -40.0)
    AND (Ta <= 100.0);

IF PropertyModelOK THEN
    HumidityRatio1 :=
        HumidityRatio(
            RelativeHumidityPct := RH1,
            TemperatureC := T1,
            PressureKPa := P1);

    HumidityRatio2 :=
        HumidityRatio(
            RelativeHumidityPct := RH2,
            TemperatureC := T2,
            PressureKPa := P2ControlRaw);

    HumidityRatioAmbient :=
        HumidityRatio(
            RelativeHumidityPct := RHa,
            TemperatureC := Ta,
            PressureKPa := Pa);

    HumidityRatioSat2 :=
        HumidityRatio(
            RelativeHumidityPct := 100.0,
            TemperatureC := T2,
            PressureKPa := P2ControlRaw);

    IF
        (HumidityRatio1 < 0.0)
        OR (HumidityRatio2 < 0.0)
```

```
        OR (HumidityRatioAmbient < 0.0)
        OR (HumidityRatioSat2 < 0.0)
    THEN
        PropertyModelOK := FALSE;
    END_IF;
END_IF;

IF PropertyModelOK THEN
    SaturationMargin :=
        HumidityRatioSat2
        -
        HumidityRatio2;

    Density1 :=
        MoistAirDensity(
            PressureKPa := P1,
            TemperatureC := T1,
            HumRatio := HumidityRatio1);

    TotalMassFlowKgS :=
        Density1
        *
        INLET_AREA_M2
        *
        U1;

    DryAirMassFlowKgS :=
        TotalMassFlowKgS
        /
        MAX(1.0 + HumidityRatio1, 0.001);

    EvapCapacityKgS :=
        DryAirMassFlowKgS
        *
        MAX(SaturationMargin, 0.0);

    LiquidCapacityRatio :=
        LiquidFlowKgS
        /
        MAX(EvapCapacityKgS, 0.000001);

ELSE
    HumidityRatio1 := 0.0;
    HumidityRatio2 := 0.0;
    HumidityRatioAmbient := 0.0;
    HumidityRatioSat2 := 0.0;
    SaturationMargin := 0.0;
    Density1 := 0.0;
    TotalMassFlowKgS := 0.0;
    DryAirMassFlowKgS := 0.0;
    EvapCapacityKgS := 0.0;
    LiquidCapacityRatio := 9999.0;
END_IF;


(************************************************************************
 8. CONSTRAINT MARGINS
************************************************************************)

PressureReserveLow :=
    P2ControlRaw
    <=
    P2Trip + PRESSURE_RESERVE_KPA;

Speed1ReserveLow :=
    N1
    >=
    N1_TRIP_RPM - N1_RESERVE_RPM;

Speed2ReserveLow :=
    N2
    >=
    N2_TRIP_RPM - N2_RESERVE_RPM;

TemperatureReserveLow :=
    (T1 >= T1_MAX_C - 10.0)
    OR
    (T2 <= T2_MIN_C + 5.0);
```

```
VibrationReserveLow :=
    (Vibration1 >= 0.90 * VIBRATION_MAX_MM_S)
    OR
    (Vibration2 >= 0.90 * VIBRATION_MAX_MM_S);

CarryoverReserveLow :=
    CarryoverKgS
    >=
    0.90 * CARRYOVER_MAX_KG_S;

SaturationReserveLow :=
    (RH2 >= RH2_MAX_PCT)
    OR
    (
        PropertyModelOK
        AND
        (SaturationMargin <= 0.0)
    )
    OR
    (
        PropertyModelOK
        AND
        (LiquidCapacityRatio >= 0.90)
    );

TurbineMapReserveLow :=
    NOT DI_TURBINE1_MAP_OK
    OR
    NOT DI_TURBINE2_MAP_OK;

ElectricalReserveLow :=
    NOT DI_ELECTRICAL_LIMITS_OK;

NearConstraint :=
    PressureReserveLow
    OR Speed1ReserveLow
    OR Speed2ReserveLow
    OR TemperatureReserveLow
    OR VibrationReserveLow
    OR CarryoverReserveLow
    OR SaturationReserveLow
    OR TurbineMapReserveLow
    OR ElectricalReserveLow;


(************************************************************************
 9. PLANT HEALTH AND TRIP CONDITIONS
************************************************************************)

CriticalSensorsOK :=
    SV_P1.Valid
    AND SV_PA.Valid
    AND SV_T1.Valid
    AND SV_T2.Valid
    AND SV_N1.Valid
    AND SV_N2.Valid
    AND PressureVoteOK;

OptimisationSensorsOK :=
    CriticalSensorsOK
    AND SV_TA.Valid
    AND SV_RH1.Valid
    AND SV_RH2.Valid
    AND SV_RHA.Valid
    AND SV_U1.Valid
    AND SV_LIQFLOW.Valid
    AND SV_VALVEPOS.Valid
    AND SV_VIB1.Valid
    AND SV_VIB2.Valid
    AND SV_CARRY.Valid
    AND PowerMeasurementsOK;

TemperatureTrip :=
    NOT DI_TEMP_PROTECTION_OK
    OR (T1 >= T1_MAX_C)
    OR (T2 <= T2_MIN_C);
```

```
VibrationTrip :=
    NOT DI_VIB_PROTECTION_OK
    OR (Vibration1 >= VIBRATION_MAX_MM_S)
    OR (Vibration2 >= VIBRATION_MAX_MM_S);

CarryoverTrip :=
    NOT DI_CARRY_PROTECTION_OK
    OR (CarryoverKgS >= CARRYOVER_MAX_KG_S);

DriveFault :=
    NOT DI_PUMP_READY
    OR NOT DI_DRIVE1_READY
    OR NOT DI_DRIVE2_READY;

CriticalSensorFault :=
    NOT CriticalSensorsOK;

WatchdogFault :=
    NOT DI_WATCHDOG_RETURN_OK;

PlantReady :=
    DI_ESTOP_OK
    AND DI_SAFETY_SYSTEM_OK
    AND DI_INDEPENDENT_TRIP_OK
    AND DI_REMOTE_IO_OK
    AND NOT DriveFault
    AND CriticalSensorsOK;

TripRequest :=
    NOT DI_ESTOP_OK
    OR NOT DI_SAFETY_SYSTEM_OK
    OR NOT DI_INDEPENDENT_TRIP_OK
    OR NOT DI_REMOTE_IO_OK
    OR P2LowTrip2oo3
    OR (N1 >= N1_TRIP_RPM)
    OR (N2 >= N2_TRIP_RPM)
    OR TemperatureTrip
    OR VibrationTrip
    OR CarryoverTrip
    OR DriveFault
    OR CriticalSensorFault
    OR WatchdogFault;


(************************************************************************
 10. DETERMINE CURRENT TRIP CAUSE
************************************************************************)

CurrentTripCause := TC_NONE;

IF NOT DI_ESTOP_OK THEN
    CurrentTripCause := TC_EMERGENCY_STOP;

ELSIF
    NOT DI_SAFETY_SYSTEM_OK
    OR NOT DI_INDEPENDENT_TRIP_OK
THEN
    CurrentTripCause := TC_INDEPENDENT_SAFETY;

ELSIF NOT DI_REMOTE_IO_OK THEN
    CurrentTripCause := TC_REMOTE_IO;

ELSIF P2LowTrip2oo3 THEN
    CurrentTripCause := TC_ZONE2_LOW_PRESSURE;

ELSIF N1 >= N1_TRIP_RPM THEN
    CurrentTripCause := TC_TURBINE1_OVERSPEED;

ELSIF N2 >= N2_TRIP_RPM THEN
    CurrentTripCause := TC_TURBINE2_OVERSPEED;

ELSIF TemperatureTrip THEN
    CurrentTripCause := TC_TEMPERATURE;

ELSIF VibrationTrip THEN
    CurrentTripCause := TC_VIBRATION;

ELSIF CarryoverTrip THEN
```

```
    CurrentTripCause := TC_LIQUID_CARRYOVER;

ELSIF DriveFault THEN
    CurrentTripCause := TC_DRIVE_OR_CONVERTER;

ELSIF CriticalSensorFault THEN
    CurrentTripCause := TC_CRITICAL_SENSOR;

ELSIF WatchdogFault THEN
    CurrentTripCause := TC_WATCHDOG;
END_IF;


(**********************************************************************
 11. TRIP LATCH AND RESTART DWELL
**********************************************************************)

SafeStateProven :=
    DI_VALVE_CLOSED
    AND DI_PUMP_STOPPED
    AND DI_BYPASS_SAFE_FB
    AND DI_GENERATOR_SAFE_FB;

ResetConditionsOK :=
    NOT TripRequest
    AND CriticalSensorsOK
    AND PressureVoteOK
    AND
    (
        P2ControlRaw
        >
        P2Set + RESET_PRESSURE_MARGIN_KPA
    )
    AND SafeStateProven;

ResetDwellTimer(
    IN := ResetConditionsOK,
    PT := RESTART_DWELL_TIME);

IF TripRequest THEN
    TripLatched := TRUE;

    IF ActiveTripCause = TC_NONE THEN
        ActiveTripCause := CurrentTripCause;
    END_IF;

ELSIF
    DI_RESET
    AND ResetDwellTimer.Q
THEN
    TripLatched := FALSE;
    ActiveTripCause := TC_NONE;
END_IF;


(**********************************************************************
 12. FAST PRESSURE LOOP
**********************************************************************)

PressurePID(
    Enable :=
        PlantReady
        AND NOT TripLatched,

    Setpoint := P2Set,
    ProcessValue := P2ControlFilt,

    Kp := KP_PRESSURE,
    Ki := KI_PRESSURE,
    Kd := KD_PRESSURE,
    Kaw := KAW_PRESSURE,
    DerivativeTau := DERIVATIVE_TAU_S,

    Ts := ControlTsS,

    OutputLo := -40.0,
    OutputHi := 40.0,
```

```
    Output => ValvePIDPct);

(*
   Provisional valve-position feedforward.
   These coefficients require process identification.
*)
(*
   Provisional valve-position feedforward.
   Feedforward is disabled if any required signal is invalid.
   Coefficients require process identification.
*)
IF
    SV_T1.Valid
    AND SV_T2.Valid
    AND SV_U1.Valid
THEN
    ValveFFPct :=
        ClampReal(
            X :=
                KFF_DELTA_T
                *
                MAX(T1 - T2, 0.0)
                +
                KFF_INLET_VELOCITY
                *
                U1,
            Lo := 0.0,
            Hi := 70.0);
ELSE
    ValveFFPct := 0.0;
END_IF;

ValveTargetPct :=
    ClampReal(
        X :=
            ValveFFPct
            +
            ValvePIDPct
            +
            ValveBiasPct,
        Lo := 0.0,
        Hi := 100.0);

PumpTargetPct :=
    ClampReal(
        X := ValveTargetPct + 5.0,
        Lo := 0.0,
        Hi := 100.0);


(*********************************************************************
 13. GENERATOR GOVERNOR TARGETS
*********************************************************************)

(*
   The command signs and gains must be proven for the actual generator,
   converter, turbine map and grid or load connection.
*)

Load1TargetPct :=
    ClampReal(
        X :=
            LOAD_BASE1_PCT
            +
            LOAD_SPEED_GAIN1
            *
            (N1 - LOAD_REFERENCE1_RPM)
            +
            Load1BiasPct,
        Lo := 10.0,
        Hi := 95.0);

Load2TargetPct :=
    ClampReal(
        X :=
            LOAD_BASE2_PCT
            +
            LOAD_SPEED_GAIN2
```

```
            *
            (N2 - LOAD_REFERENCE2_RPM)
            +
            Load2BiasPct,
        Lo := 10.0,
        Hi := 95.0);


(************************************************************************
 14. CONSTRAINED COMMAND PROJECTION
************************************************************************)

(*
   When Zone 2 pressure reserve is low, the projection prevents an
   increase in spray while the fast pressure loop reduces injection.
*)

IF PressureReserveLow THEN
    ValveTargetPct :=
        MIN(
            ValveTargetPct,
            ValveCmdPct);

    PumpTargetPct :=
        MIN(
            PumpTargetPct,
            PumpCmdPct);
END_IF;

ValveTargetPct :=
    ClampReal(
        X := ValveTargetPct,
        Lo := 0.0,
        Hi := 100.0);

PumpTargetPct :=
    ClampReal(
        X := PumpTargetPct,
        Lo := 0.0,
        Hi := 100.0);

Load1TargetPct :=
    ClampReal(
        X := Load1TargetPct,
        Lo := 0.0,
        Hi := 100.0);

Load2TargetPct :=
    ClampReal(
        X := Load2TargetPct,
        Lo := 0.0,
        Hi := 100.0);


(************************************************************************
 15. OPTIMISER PERMISSION AND SETTLING CONDITIONS
************************************************************************)

AllActuatorsTracking :=
    DI_VALVE_TRACKING
    AND DI_PUMP_TRACKING
    AND DI_LOAD1_TRACKING
    AND DI_LOAD2_TRACKING;

ProcessSettled :=
    ABS(P2ControlFilt - P2Set)
        <= PRESSURE_CONTROL_BAND_KPA
    AND ABS(N1RateRPMs)
        <= SPEED_SETTLED_RATE_RPM_S
    AND ABS(N2RateRPMs)
        <= SPEED_SETTLED_RATE_RPM_S
    AND DI_DRIVES_STABLE
    AND AllActuatorsTracking
    AND NOT NearConstraint;

OptEnabled :=
    PlantReady
    AND TaskTimeOK
```

```
    AND NOT TripLatched
    AND OptimisationSensorsOK
    AND PropertyModelOK
    AND NOT PressureVoteDegraded
    AND NOT NearConstraint;


(***********************************************************************
 16. OPTIMISER TIMERS
***********************************************************************)

BaselineTimer(
    IN :=
        OptEnabled
        AND (OptState = OPT_BASELINE)
        AND ProcessSettled,
    PT := OPT_BASELINE_TIME);

ActuatorTimer(
    IN :=
        OptEnabled
        AND (OptState = OPT_WAIT_ACTUATOR)
        AND AllActuatorsTracking,
    PT := OPT_ACTUATOR_TIME);

SettlingTimer(
    IN :=
        OptEnabled
        AND (OptState = OPT_WAIT_SETTLE)
        AND ProcessSettled,
    PT := OPT_SETTLE_TIME);

TrialSampleTimer(
    IN :=
        OptEnabled
        AND (OptState = OPT_AVERAGE_TRIAL)
        AND ProcessSettled,
    PT := OPT_SAMPLE_TIME);

OptimiserTimeoutTimer(
    IN :=
        OptEnabled
        AND (OptState <> OPT_BASELINE),
    PT := OPT_TIMEOUT_TIME);


(***********************************************************************
 17. BOUNDED PERTURB-AND-OBSERVE OPTIMISER
***********************************************************************)

IF NOT OptEnabled THEN
    (*
       Roll back an incomplete trial whenever optimisation permission
       is lost.
    *)
    IF OptState <> OPT_BASELINE THEN
        CASE OptIndex OF
            0:
                ValveBiasPct := OldBiasPct;

            1:
                Load1BiasPct := OldBiasPct;

            2:
                Load2BiasPct := OldBiasPct;
        END_CASE;
    END_IF;

    OptState := OPT_BASELINE;
    BaselinePowerSum := 0.0;
    TrialPowerSum := 0.0;
    BaselineSampleCount := 0;
    TrialSampleCount := 0;

ELSE
    CASE OptState OF

        OPT_BASELINE:
```

```
    IF ProcessSettled THEN
        BaselinePowerSum :=
            BaselinePowerSum
            +
            REAL_TO_LREAL(PnetFilt);

        BaselineSampleCount :=
            BaselineSampleCount + 1;

    ELSE
        BaselinePowerSum := 0.0;
        BaselineSampleCount := 0;
    END_IF;

    IF BaselineTimer.Q THEN
        IF BaselineSampleCount > 0 THEN
            BaselinePowerAvg :=
                LREAL_TO_REAL(
                    BaselinePowerSum
                    /
                    DINT_TO_LREAL(BaselineSampleCount));

            OptState := OPT_APPLY_TRIAL;
        END_IF;
    END_IF;


OPT_APPLY_TRIAL:
    CASE OptIndex OF
        0:
            OldBiasPct := ValveBiasPct;

            ValveBiasPct :=
                ClampReal(
                    X :=
                        ValveBiasPct
                        +
                        OptDirection[0]
                        *
                        TrialStepPct,
                    Lo := -OPT_MAX_BIAS_PCT,
                    Hi := OPT_MAX_BIAS_PCT);

        1:
            OldBiasPct := Load1BiasPct;

            Load1BiasPct :=
                ClampReal(
                    X :=
                        Load1BiasPct
                        +
                        OptDirection[1]
                        *
                        TrialStepPct,
                    Lo := -OPT_MAX_BIAS_PCT,
                    Hi := OPT_MAX_BIAS_PCT);

        2:
            OldBiasPct := Load2BiasPct;

            Load2BiasPct :=
                ClampReal(
                    X :=
                        Load2BiasPct
                        +
                        OptDirection[2]
                        *
                        TrialStepPct,
                    Lo := -OPT_MAX_BIAS_PCT,
                    Hi := OPT_MAX_BIAS_PCT);
    END_CASE;

    TrialPowerSum := 0.0;
    TrialSampleCount := 0;
    OptState := OPT_WAIT_ACTUATOR;


OPT_WAIT_ACTUATOR:
```

```
    IF ActuatorTimer.Q THEN
        OptState := OPT_WAIT_SETTLE;
    END_IF;


OPT_WAIT_SETTLE:
    IF SettlingTimer.Q THEN
        TrialPowerSum := 0.0;
        TrialSampleCount := 0;
        OptState := OPT_AVERAGE_TRIAL;
    END_IF;


OPT_AVERAGE_TRIAL:
    IF ProcessSettled THEN
        TrialPowerSum :=
            TrialPowerSum
            +
            REAL_TO_LREAL(PnetFilt);

        TrialSampleCount :=
            TrialSampleCount + 1;
    END_IF;

    IF TrialSampleTimer.Q THEN
        IF TrialSampleCount > 0 THEN
            TrialPowerAvg :=
                LREAL_TO_REAL(
                    TrialPowerSum
                    /
                    DINT_TO_LREAL(TrialSampleCount));
        ELSE
            TrialPowerAvg := -1.0E30;
        END_IF;

        OptState := OPT_DECIDE;
    END_IF;


OPT_DECIDE:
    IF
        (
            TrialPowerAvg
            >
            BaselinePowerAvg
            +
            POWER_IMPROVEMENT_MIN_KW
        )
        AND
        NOT NearConstraint
        AND
        ProcessSettled
    THEN
        (*
           Accept the improved feasible point.
           The perturbed bias remains in service.
        *)
        BaselinePowerAvg := TrialPowerAvg;

    ELSE
        (*
           Reject and roll back the trial.
        *)
        CASE OptIndex OF
            0:
                ValveBiasPct := OldBiasPct;
                OptDirection[0] :=
                    -OptDirection[0];

            1:
                Load1BiasPct := OldBiasPct;
                OptDirection[1] :=
                    -OptDirection[1];

            2:
                Load2BiasPct := OldBiasPct;
                OptDirection[2] :=
                    -OptDirection[2];
```

```
                END_CASE;

                TrialStepPct :=
                    MAX(
                        OPT_MIN_STEP_PCT,
                        0.80 * TrialStepPct);
            END_IF;

            OptIndex := OptIndex + 1;

            IF OptIndex > 2 THEN
                OptIndex := 0;
            END_IF;

            BaselinePowerSum := 0.0;
            TrialPowerSum := 0.0;
            BaselineSampleCount := 0;
            TrialSampleCount := 0;

            OptState := OPT_BASELINE;

    END_CASE;


    (*
       Abort and roll back a trial that cannot obtain actuator proof
       or a stable measurement window within the allowed timeout.
    *)
    IF OptimiserTimeoutTimer.Q THEN
        CASE OptIndex OF
            0:
                ValveBiasPct := OldBiasPct;
                OptDirection[0] :=
                    -OptDirection[0];

            1:
                Load1BiasPct := OldBiasPct;
                OptDirection[1] :=
                    -OptDirection[1];

            2:
                Load2BiasPct := OldBiasPct;
                OptDirection[2] :=
                    -OptDirection[2];
        END_CASE;

        TrialStepPct :=
            MAX(
                OPT_MIN_STEP_PCT,
                0.80 * TrialStepPct);

        BaselinePowerSum := 0.0;
        TrialPowerSum := 0.0;
        BaselineSampleCount := 0;
        TrialSampleCount := 0;

        OptState := OPT_BASELINE;
    END_IF;
END_IF;


(*******************************************************************
 18. RATE-LIMIT NORMAL COMMANDS
*******************************************************************)

ValveCmdPct :=
    RateLimit(
        Target := ValveTargetPct,
        Previous := ValveCmdPct,
        RisePerS := 20.0,
        FallPerS := 40.0,
        Ts := ControlTsS);

PumpCmdPct :=
    RateLimit(
        Target := PumpTargetPct,
        Previous := PumpCmdPct,
        RisePerS := 20.0,
```

```
        FallPerS := 40.0,
        Ts := ControlTsS);

Load1CmdPct :=
    RateLimit(
        Target := Load1TargetPct,
        Previous := Load1CmdPct,
        RisePerS := 10.0,
        FallPerS := 20.0,
        Ts := ControlTsS);

Load2CmdPct :=
    RateLimit(
        Target := Load2TargetPct,
        Previous := Load2CmdPct,
        RisePerS := 10.0,
        FallPerS := 20.0,
        Ts := ControlTsS);


(***********************************************************************
 19. CAUSE-SPECIFIC SAFE GENERATOR COMMANDS
***********************************************************************)

SafeLoad1Pct := SAFE_PROCESS_LOAD1_PCT;
SafeLoad2Pct := SAFE_PROCESS_LOAD2_PCT;
SafeLoadEnable := SAFE_PROCESS_LOAD_ENABLE;

CASE ActiveTripCause OF

    TC_TURBINE1_OVERSPEED,
    TC_TURBINE2_OVERSPEED:

        SafeLoad1Pct := SAFE_OVERSPEED_LOAD1_PCT;
        SafeLoad2Pct := SAFE_OVERSPEED_LOAD2_PCT;
        SafeLoadEnable := SAFE_OVERSPEED_LOAD_ENABLE;

    TC_NONE:
        SafeLoad1Pct := SAFE_PROCESS_LOAD1_PCT;
        SafeLoad2Pct := SAFE_PROCESS_LOAD2_PCT;
        SafeLoadEnable := SAFE_PROCESS_LOAD_ENABLE;

ELSE
    SafeLoad1Pct := SAFE_PROCESS_LOAD1_PCT;
    SafeLoad2Pct := SAFE_PROCESS_LOAD2_PCT;
    SafeLoadEnable := SAFE_PROCESS_LOAD_ENABLE;
END_CASE;


(***********************************************************************
 20. SAFE-STATE OVERRIDE
***********************************************************************)

IF TripLatched THEN
    ValveCmdPct := 0.0;
    PumpCmdPct := 0.0;

    Load1CmdPct :=
        ClampReal(
            X := SafeLoad1Pct,
            Lo := 0.0,
            Hi := 100.0);

    Load2CmdPct :=
        ClampReal(
            X := SafeLoad2Pct,
            Lo := 0.0,
            Hi := 100.0);

    DO_PUMP_ENABLE := FALSE;
    DO_LOAD_ENABLE := SafeLoadEnable;
    DO_BYPASS_SAFE := TRUE;
    DO_TRIP_RELAY := TRUE;

ELSE
    DO_PUMP_ENABLE := PumpCmdPct > 1.0;
    DO_LOAD_ENABLE := TRUE;
    DO_BYPASS_SAFE := FALSE;
```

```
    DO_TRIP_RELAY := FALSE;
END_IF;


(**********************************************************************
 21. FINAL-ELEMENT PROOF
**********************************************************************)

SafeProofTimer(
    IN :=
        TripLatched
        AND NOT SafeStateProven,
    PT := SAFE_PROOF_TIME);

IF SafeProofTimer.Q THEN
    FinalElementFaultLatched := TRUE;
END_IF;

IF
    DI_RESET
    AND ResetDwellTimer.Q
THEN
    FinalElementFaultLatched := FALSE;
END_IF;

FinalElementFault := FinalElementFaultLatched;

DegradedOperation :=
    NOT TaskTimeOK
    OR PressureVoteDegraded
    OR NOT PropertyModelOK
    OR NOT OptimisationSensorsOK;

DO_ALARM :=
    TripLatched
    OR DegradedOperation
    OR FinalElementFault;


(**********************************************************************
 22. WATCHDOG OUTPUT
**********************************************************************)

WatchdogTimer(
    IN := NOT WatchdogReset,
    PT := WATCHDOG_HALF_PERIOD);

IF WatchdogTimer.Q THEN
    WatchdogBit := NOT WatchdogBit;
    WatchdogReset := TRUE;

ELSIF WatchdogReset THEN
    (* Give the TON one scan with IN = FALSE before restarting it. *)
    WatchdogReset := FALSE;
END_IF;

DO_WATCHDOG := WatchdogBit;


(**********************************************************************
 23. WRITE PHYSICAL OUTPUTS
**********************************************************************)

AO_VALVE_RAW :=
    ToAO4_20mA(
        Percent := ValveCmdPct);

AO_PUMP_RAW :=
    ToAO4_20mA(
        Percent := PumpCmdPct);

AO_LOAD1_RAW :=
    ToAO4_20mA(
        Percent := Load1CmdPct);

AO_LOAD2_RAW :=
    ToAO4_20mA(
        Percent := Load2CmdPct);
```

```
(*******************************************************************
 24. END-OF-SCAN STATE UPDATES
*******************************************************************)

PreviousN1 := N1;
PreviousN2 := N2;
FirstScan := FALSE;

END_PROGRAM
```

The code intentionally exposes the atmospheric pressure factors, normal pressure setpoint, protective trip factor, sensor ranges, rate-of-change limits, pressure-voting tolerance, filter time constants, PID gains, actuator rate limits, turbine-speed limits, optimisation step size, settling and averaging periods, restart dwell, final-element proof time, and cause-specific safe generator commands as commissioning parameters. These values must not be transferred to a physical installation without validated process identification, turbine and generator studies, actuator testing, measurement-uncertainty assessment, hazard and operability analysis, machinery risk assessment, and confirmation of the direction, magnitude and dynamic response of every controlled variable. Independent emergency-stop, overspeed and critical-pressure protection must remain separate from the optimisation controller.

51.10 Recommended controller platforms and I/O

The final controller should be selected only after confirming scan-time, high-speed-counter, turbine-governor, safety-integrity, environmental, cybersecurity, communications and maintainability requirements. The table lists practical starting points rather than interchangeable equivalents.

Operational condition	Recommended processor and part number	Recommended I/O	Application rationale
Laboratory thermodynamic rig and high-speed control development	National Instruments cRIO-9045, p/n 785623-01	NI-9203 current input, p/n 779516-01; NI-9265 current output, p/n 779334-01; NI-9213 thermocouple input, p/n 785185-01	FPGA-assisted acquisition, deterministic control, data logging and rapid algorithm development; not a replacement for a separate certified trip system.[21]
Pilot or industrial non-hazardous installation with two coordinated turbines	Siemens SIMATIC S7-1500T CPU 1515T-2 PN, p/n 6ES7515-2TN03-0AB0	ET 200SP AI 4xI 2-/4-wire ST, p/n 6ES7134-6GD01-0BA1; ET 200SP AQ 4xU/I ST, p/n 6ES7135-6HD00-0BA1	Integrated motion/technology functions, PROFINET and maintainable industrial architecture.[22][23]
Distributed outdoor or high-vibration skid, non-safety edge control	WAGO PFC200 Extreme, p/n 750-8212/040-010	WAGO 750-series isolated analogue and digital I/O selected for the signal type	Extended-environment controller with M12 Ethernet and serial interfaces; use a separate safety layer where required.[24]
EtherCAT-based research or machinery platform	Beckhoff CX5140 Embedded PC	Beckhoff EL3xxx analogue inputs, EL4xxx analogue outputs and EL5101/EL5151 high-speed counters selected to signal type	Suitable where high-rate EtherCAT acquisition, PC-based control and model integration are priorities.[25]
Hazardous or higher-integrity process installation	Siemens SIMATIC S7-1500F CPU 1515F-2 PN, p/n 6ES7515-2FN03-0AB0, with a separately engineered safety application	Safety-rated and intrinsically safe remote I/O/barriers selected for the area classification	Fail-safe CPU option for a formally designed safety lifecycle; it does not by itself establish a required SIL or machinery performance level.[26][27]

51.11 Recommended instrumentation and final-control parts by operating condition

Process instruments, control valves and dosing pumps are highly configurable. For those products, the model or series is the recommended part number; the complete ordering code must be generated from the actual measurement range, pressure class, maximum gas, liquid and metal temperature, wetted material, process connection, electrical output, ingress protection and hazardous-area approval. The selection shall also confirm thermowell response, valve and nozzle suitability for flashing or cavitating service, pump net positive suction head under the hottest liquid condition, seal and elastomer compatibility, vapour or two-phase-flow suitability, and drainage and carryover detection requirements. A configured ordering code for one condition must not be copied to another.

[21] National Instruments (2026) 'cRIO-9045, NI-9203, NI-9265 and NI-9213 product information'. Austin, TX: NI. Available at: https://www.ni.com/ (Accessed: 26 June 2026).
[22] Siemens (2026) 'SIMATIC S7-1500T CPU 1515T-2 PN, article number 6ES7515-2TN03-0AB0'. Munich: Siemens Industry Mall. Available at: https://mall.industry.siemens.com/ (Accessed: 26 June 2026).
[23] Siemens (2026) 'SIMATIC ET 200SP AI 4xI 2-/4-wire ST, 6ES7134-6GD01-0BA1, and AQ 4xU/I ST, 6ES7135-6HD00-0BA1'. Munich: Siemens Industry Mall. Available at: https://mall.industry.siemens.com/ (Accessed: 26 June 2026).
[24] WAGO (2026) 'Controller PFC200 Extreme, item 750-8212/040-010'. Minden: WAGO. Available at: https://www.wago.com/ (Accessed: 26 June 2026).
[25] Beckhoff Automation (2026) 'CX5140 Embedded PC with Intel Atom E3845'. Verl: Beckhoff Automation. Available at: https://www.beckhoff.com/ (Accessed: 26 June 2026).
[26] Siemens (2026) 'SIMATIC S7-1500F CPU 1515F-2 PN, article number 6ES7515-2FN03-0AB0'. Munich: Siemens Industry Mall. Available at: https://mall.industry.siemens.com/ (Accessed: 26 June 2026).
[27] International Electrotechnical Commission (2016) *IEC 61511-1:2016+AMD1:2017 Functional safety—Safety instrumented systems for the process industry sector—Part 1: Framework, definitions, system, hardware and application programming requirements*. Geneva: IEC.

51.11.1 Condition A — clean-air laboratory or low-pressure prototype

Representative envelope: clean non-hazardous air, approximately 0–50 °C, near-atmospheric pressure, low liquid-flow rate and accessible manual supervision.

Function	Manufacturer and recommended part/model	Interface	Selection note
Controller	NI cRIO-9045, p/n 785623-01	Ethernet, RS-232/485 and C Series I/O	Preferred for high-rate test data and controller development.
Current input/output	NI-9203 p/n 779516-01; NI-9265 p/n 779334-01	±20 mA input; current output	Provides real 4–20 mA sensor and actuator connection.
Temperature input	NI-9213 p/n 785185-01	Thermocouple input	Use with grounded/ungrounded junction selection appropriate to the rig.
Pressure	WIKA A-10 pressure transmitter, configured order code; low-range example article 52811886 is 0–50 inWC gauge	4–20 mA	Select absolute transmitters for P_1, P_2 and P_a when pressure ratios are calculated.[28]
Temperature	WIKA TR10-B Pt100 assembly; example 110 mm article 14426962 where geometry is suitable	Pt100 or head-mounted 4–20 mA transmitter	Final insertion length and thermowell must be checked.[29]
Ambient/inlet humidity and atmosphere	Vaisala HMP7 probe with Indigo520 transmitter and barometric-pressure module	Analogue outputs, Modbus TCP or configurable digital interfaces	Useful for high-humidity measurement and combined atmospheric pressure, humidity and temperature acquisition.[30]
Velocity	DwyerOmega Series 641RM remote-probe air-velocity transmitter	4–20 mA	Appropriate only for clean air and within the probe's stated range.[31]
Fine liquid valve	Bürkert Type 2873; example configured item 234289 for a small 0.8 mm, 24 V configuration	PWM/current driver according to selected version	Direct-acting proportional valve for small clean-liquid flows; select seal, body and orifice for the injected liquid.[32]
Dosing pump	Grundfos DDA SMART Digital, configured size	Digital/analogue or fieldbus options by configuration	Select through the manufacturer's dosing tool from required flow, pressure and material compatibility.[33]

[28] WIKA (2026) 'Model A-10 pressure transmitter'. Klingenberg: WIKA. Available at: https://www.wika.com/ (Accessed: 26 June 2026).
[29] WIKA (2026) 'Model TR10-B resistance thermometer'. Klingenberg: WIKA. Available at: https://www.wika.com/ (Accessed: 26 June 2026).
[30] Vaisala (2026) 'HMP7 relative humidity and temperature probe and Indigo520 transmitter with barometric pressure measurement'. Vantaa: Vaisala. Available at: https://www.vaisala.com/ (Accessed: 26 June 2026).
[31] DwyerOmega (2026) 'Series 641RM air velocity transmitter with remote probe'. Michigan City, IN: DwyerOmega. Available at: https://www.dwyeromega.com/ (Accessed: 26 June 2026).
[32] Bürkert (2026) 'Type 2873 direct-acting 2-way proportional valve'. Ingelfingen: Christian Bürkert GmbH & Co. KG. Available at: https://www.burkert.com/ (Accessed: 26 June 2026).
[33] Grundfos (2026) 'DDA SMART Digital dosing pump and dosing-pump selection tool'. Bjerringbro: Grundfos. Available at: https://www.grundfos.com/ (Accessed: 26 June 2026).

51.11.2 Condition B — pilot or industrial clean-gas system

Representative envelope: non-hazardous clean or moderately wet gas, approximately -20–120 °C, pressures up to the selected transmitter and valve ratings, continuous automatic operation.

Function	Manufacturer and recommended part/model	Interface	Selection note
Controller	Siemens CPU 1515T-2 PN, p/n 6ES7515-2TN03-0AB0	PROFINET IRT/RT	Main industrial regulator and coordinated turbine/loading control.
Pressure	Emerson Rosemount 3051S Pressure Transmitter family	4–20 mA/HART, WirelessHART or configured digital option	Configure absolute pressure ranges for Zone 1, Zone 2 and atmosphere; use three independent Zone 2 transmitters where the risk analysis requires voting.[34]
Temperature	Emerson Rosemount 3144P transmitter with suitable dual-element Pt100 sensor and thermowell	4–20 mA/HART	High-accuracy process temperature measurement with sensor diagnostics; complete model code is application-specific.[35]
Humidity/ambient	Vaisala HMP7 with Indigo520 and pressure module	Analogue or Modbus TCP	Use probe heating/condensation protection when required.
Gas mass flow	Fluid Components International ST80/ST80L	4–20 mA and configured digital communications	Industrial thermal-dispersion air/gas flow meter; select the application-matched or wet-gas sensing element.[36]
Liquid control valve	Bürkert Type 3360 electromotive angle-seat control valve; example configured item **291061**	24 V DC with selected analogue/digital communications	Suitable for larger modulating liquid flow than the Type 2873; size by required C_v, pressure drop and material.[37]
Dosing pump	Grundfos DDA SMART Digital, configured size	Analogue/digital/fieldbus by configuration	Use for moderate-pressure clean-liquid dosing.
Generator/motor drive	ABB ACS580-01 or Siemens SINAMICS G120, size/configuration selected to machine	Industrial Ethernet/fieldbus and hardwired safe controls	Drive family must be selected from generator topology, voltage, current, regeneration and braking requirements.[38]

[34] Emerson (2026) 'Rosemount 3051S pressure transmitter family'. St Louis, MO: Emerson. Available at: https://www.emerson.com/ (Accessed: 26 June 2026).
[35] Emerson (2026) 'Rosemount 3144P temperature transmitter'. St Louis, MO: Emerson. Available at: https://www.emerson.com/ (Accessed: 26 June 2026).
[36] Fluid Components International (2026) 'ST80/ST80L mass flow meters'. San Marcos, CA: Fluid Components International. Available at: https://www.fluidcomponents.com/ (Accessed: 26 June 2026).
[37] Bürkert (2026) 'Type 3360 electromotive 2-way angle-seat control valve'. Ingelfingen: Christian Bürkert GmbH & Co. KG. Available at: https://www.burkert.com/ (Accessed: 26 June 2026).
[38] ABB (2026) 'ACS580-01 wall-mounted drives'. Zurich: ABB; Siemens (2026) 'SINAMICS G120 modular frequency converter'. Munich: Siemens. Available at: https://www.abb.com/ and https://www.siemens.com/ (Accessed: 26 June 2026).

51.11.3 Condition C — hot, wet or particulate industrial exhaust

Representative envelope: approximately 120–450 °C gas, large ducts, condensable vapour, possible particulate loading and higher actuator forces. Instruments must be isolated from temperatures or contamination beyond their ratings.

Function	Manufacturer and recommended part/model	Interface	Selection note
Controller	Siemens CPU 1515T-2 PN in a conditioned enclosure; WAGO PFC200 Extreme 750-8212/040-010 may be used for remote non-safety acquisition	PROFINET or industrial Ethernet	Keep the controller and standard I/O within their specified ambient environment.
Pressure	Rosemount 3051S family with remote seals, condensate arrangements or engineered impulse lines as required	4–20 mA/HART	Prevent hot, dirty or condensing gas from directly damaging the sensing cell.
Temperature	Endress+Hauser iTHERM ModuLine TM131 RTD/TC assembly with application-specific thermowell	Head transmitter, 4–20 mA/HART or configured protocol	Modular industrial thermometer available as RTD or thermocouple; select sheath and thermowell for velocity, corrosion and temperature.[39]
Humidity	Extractive, filtered and temperature-conditioned sample system feeding Vaisala HMP7/Indigo520	Digital or analogue	Do not expose the humidity probe directly beyond its temperature, pressure, contamination or condensation limits.
Gas velocity/volume flow	SICK/Endress+Hauser FLOWSIC100; use the appropriate Process/AC or Flare-XT variant	Analogue and digital outputs by configuration	Ultrasonic measurement avoids an intrusive moving rotor; AC variants are available for gas temperatures up to 450 °C according to the manufacturer.[40]
Process control valve	Fisher control valve with FIELDVUE DVC6200 digital valve controller	4–20 mA/HART, FOUNDATION Fieldbus or PROFIBUS option	Valve body, actuator, trim and failure action must be selected by a control-valve sizing study.[41]
High-pressure metering pump	LEWA ecoflow diaphragm metering pump, configured pump head and drive	Local/remote control according to package	Modular industrial metering-pump family for higher-pressure or demanding fluids.[42]
Generator drive/load	SINAMICS G120 or ABB ACS580-01 only where compatible with the electrical machine; otherwise a purpose-designed active front end and dump-load system	Fieldbus plus independent protective signals	Regeneration and loss-of-grid behaviour require a dedicated electrical study.

[39] Endress+Hauser (2025) 'iTHERM ModuLine TM131 industrial modular RTD/TC thermometer'. Reinach: Endress+Hauser. Available at: https://www.endress.com/ (Accessed: 26 June 2026).
[40] SICK (2026) 'FLOWSIC100 ultrasonic gas flow measuring system'. Waldkirch: SICK. Available at: https://www.sick.com/ (Accessed: 26 June 2026).
[41] Emerson (2026) 'Fisher FIELDVUE DVC6200 digital valve controller'. Marshalltown, IA: Emerson. Available at: https://www.emerson.com/ (Accessed: 26 June 2026).
[42] LEWA (2026) 'LEWA ecoflow diaphragm metering pump'. Leonberg: LEWA GmbH. Available at: https://www.lewa.com/ (Accessed: 26 June 2026).

51.11.4 Condition D — hazardous area or safety-critical process

Representative envelope: classified gas or dust area, flammable process, high consequence of pressure loss or overspeed, and a formally assigned safety-integrity requirement.

Function	Manufacturer and recommended part/model	Interface	Selection note
Control and safety processor	Siemens CPU 1515F-2 PN, p/n 6ES7515-2FN03-0AB0, with segregated standard and safety programs	PROFINET/PROFIsafe	Final architecture must follow the safety-requirements specification and verification plan.
Zone 2 pressure	Three Rosemount 3051S transmitters in approved hazardous-area configurations, or another independently certified transmitter set	4–20 mA/HART through approved barriers or suitable fieldbus	Use independent process connections where common-cause blockage is credible.
Temperature	Endress+Hauser iTHERM TM131 in the required intrinsically safe/explosion-protected configuration	Approved transmitter/barrier system	Complete ordering code depends on area classification and protection concept.
Humidity	Vaisala HMT370EX series, probe selected to gas and mounting condition	Intrinsically safe transmitter system	Designed for humidity and temperature measurement in hazardous environments; final probe and approvals must match the zone and gas group.[43]
Gas flow	FLOWSIC100 Flare-XT or another certified flow system selected for the process and area	Certified analogue/digital interfaces	Confirm gas composition, pressure, temperature, required straight lengths and certification.
Shutdown/control valve	Fisher process valve with FIELDVUE DVC6200 SIS where suitable	Safety-rated interface and partial-stroke diagnostics as engineered	The positioner is only one element of the complete final element and safety function.[44]
Metering pump	LEWA ecoflow package with suitable hazardous-area motor, instrumentation and materials	Package-specific	Select through a process and hazardous-area engineering review.
Independent protection	Separate overspeed device, hardwired emergency stop, certified barriers/isolators, safe power isolation and appropriately designed bypass/relief path	Safety-rated hardwired and networked signals	The optimisation PLC must not be the sole layer preventing overspeed, reverse flow, overpressure or loss of containment.

[43] Vaisala (2026) 'HMT370EX intrinsically safe humidity and temperature transmitter series'. Vantaa: Vaisala. Available at: https://www.vaisala.com/ (Accessed: 26 June 2026).
[44] Emerson (2026) 'Fisher FIELDVUE DVC6200 SIS digital valve controller'. Marshalltown, IA: Emerson. Available at: https://www.emerson.com/ (Accessed: 26 June 2026).

51.12 Commissioning and verification requirements

Before optimisation is enabled, the following tests are required:

1. Calibrate every pressure, temperature, humidity, velocity, speed and electrical-power channel through the complete installed loop.
1. Confirm that absolute and gauge pressures are not mixed in the pressure-factor calculation.
2. Prove analogue underrange, overrange, open-circuit, frozen-value and disagreement diagnostics.
3. Conduct valve stroke, pump minimum-flow, actuator deadband, hysteresis and fail-position tests.
4. Identify the sign and dynamic gain from liquid injection to Zone 2 pressure and temperature at several safe operating points.
5. Identify the sign and dynamic gain from generator loading to turbine speed, pressure and electrical output.
6. Establish the minimum safe pressure reserve and trip factor from transient tests and validated models, not from an arbitrary percentage.
7. Test loss of power, loss of network, loss of field I/O, sensor failure, valve failure, pump failure, generator trip, grid loss, turbine overspeed, outlet backflow, excessive liquid-supply temperature, invalid or unavailable liquid-property calculation, operation at or above the declared critical-pressure boundary, unexpected flash generation, pump cavitation, vapour locking, thermal shock, excessive vapour loading and unstable evaporation-regime switching scenarios.
8. Verify the independent emergency shutdown and overspeed paths before enabling automatic operation.
9. Run the fast loops with the optimiser disabled until stability and constraint margins are demonstrated.
10. Enable one optimisation coordinate at a time with small perturbations, long settling periods and conservative acceptance thresholds.
11. Validate net-power improvement using calibrated electrical power meters and an energy balance over a sufficiently long period.
12. Retain an audit log of controller version, calibration constants, limits, optimisation decisions, operator actions and all trips.
13. Freeze the validated production version under formal management of change.

The controller software should be developed and documented in accordance with the applicable programmable-controller and machinery electrical standards.[45] Where the installation falls within the process-industry functional-safety domain, the safety lifecycle, independence, proof testing and competence requirements must be addressed under the applicable edition of IEC 61511 and local legislation.[46]

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[45] International Electrotechnical Commission (2016) *IEC 60204-1:2016+AMD1:2021 Safety of machinery—Electrical equipment of machines—Part 1: General requirements*. Geneva: IEC.
[46] International Electrotechnical Commission (2016) *IEC 61511-1:2016+AMD1:2017 Functional safety—Safety instrumented systems for the process industry sector—Part 1: Framework, definitions, system, hardware and application programming requirements*. Geneva: IEC.

Annexure A

US Patent 12,650,086 B1: Device for Conversion of Kinetic and Thermal Energies of Gas Flows to Work

US012650086B1

(12) United States Patent
de St. Amatus et al.

(10) Patent No.: US 12,650,086 B1
(45) Date of Patent: Jun. 9, 2026

(54) **DEVICE FOR CONVERSION OF KINETIC AND THERMAL ENERGIES OF GAS FLOWS TO WORK**

(71) Applicants: **Benedict de St. Amatus**, Forest Glen (AU); **Alexander de St. Amatus**, Forest Glen (AU)

(72) Inventors: **Benedict de St. Amatus**, Forest Glen (AU); **Alexander de St. Amatus**, Forest Glen (AU)

(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 0 days.

(21) Appl. No.: **19/195,430**

(22) Filed: **Apr. 30, 2025**

(51) **Int. Cl.**

F03D 1/02	(2006.01)
F01D 21/00	(2006.01)
F01D 25/30	(2006.01)

(52) **U.S. Cl.**

CPC ***F01D 25/305*** (2013.01); ***F01D 21/003*** (2013.01); *F05D 2260/212* (2013.01); *F05D 2270/301* (2013.01); *F05D 2270/303* (2013.01)

(58) **Field of Classification Search**

CPC ... F03D 1/02; F03D 1/025; F03D 1/04; F03D 1/041; F03D 1/042; F03D 1/044; F03D 1/051; F03D 1/053; F03D 7/00; F03D 9/00; F03D 9/007; F03D 9/008; F03D 9/14; F03D 9/18; F03D 9/28; F03D 9/35; F03D 9/37; F03D 9/39; F03D 80/60; F03D 80/601; F03D 9/41; F03D 17/018; F03D 17/009; F03D 17/022; F03D 17/026; F01D 25/305; F01D 21/003; F05D 2260/212; F05D 2270/301; F05D 2270/3011; F05D 2270/3013; F05D 2270/3015; F05D 2270/303

See application file for complete search history.

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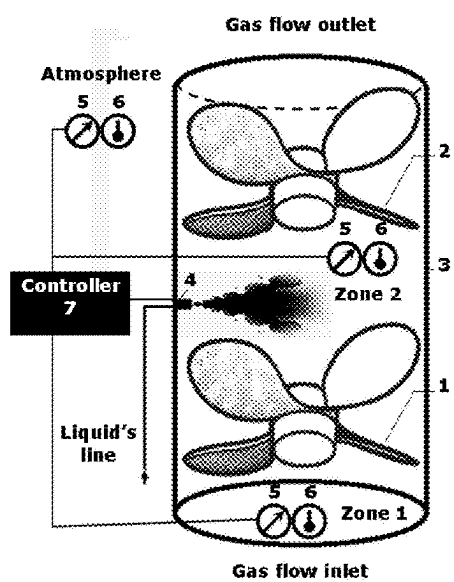
Primary Examiner — Nathaniel E Wiehe
Assistant Examiner — Joshua R Beebe

(57) **ABSTRACT**

An energy conversion device transforms kinetic and thermal energies of gas flows into usable work. It features a duct with an inlet and outlet for the gas flow and a cascade of at least two turbines located in distinct zones of the duct. A post-turbine gas cooling system, associated with the first turbine, includes an injector of evaporating liquid regulated by a controller using real-time data from pressure and temperature gauges. The first turbine's performance is enhanced by pressure and temperature differentials created by the cooling system. The second turbine prevents ambient air intrusion and further converts residual kinetic energy. Both turbines can be coupled either to electric generators or to mechanical devices or both for converting the turbines work output either to electricity or to mechanical work or to both.

3 Claims, 1 Drawing Sheet



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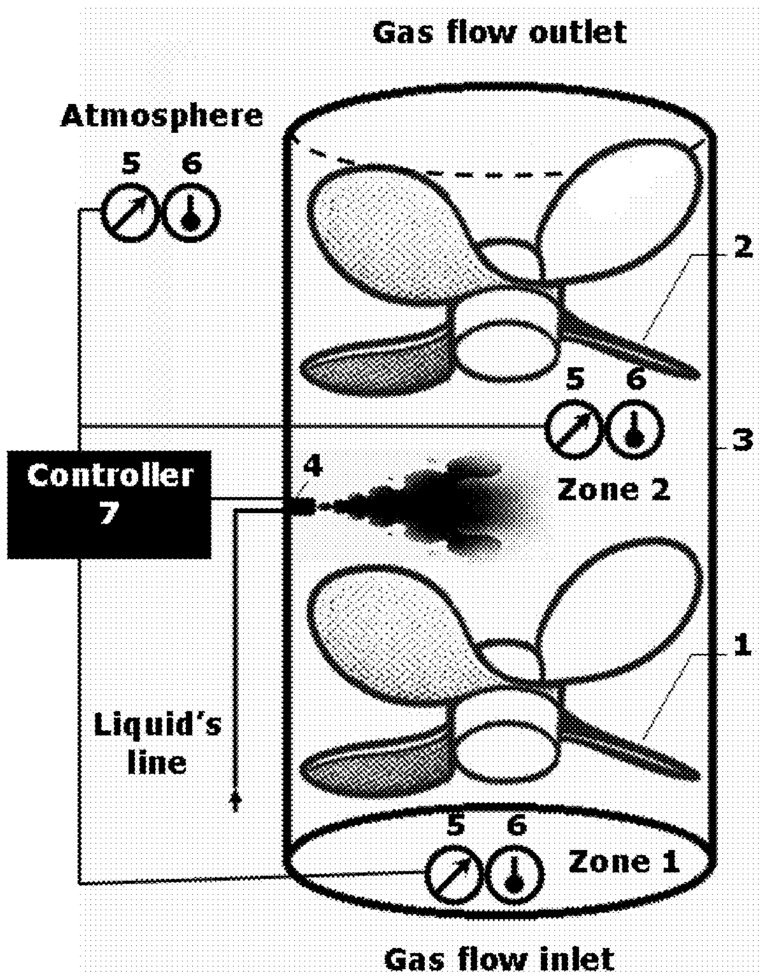
Gas flow outlet
Atmosphere
5
6
2
5
6
3
Controller
7
4
Zone 2
1
Liquid's
line
5
6
Zone 1
Gas flow inlet

DEVICE FOR CONVERSION OF KINETIC AND THERMAL ENERGIES OF GAS FLOWS TO WORK

CROSS-REFERENCE TO RELATED APPLICATIONS

Not Applicable

STATEMENT REGARDING FEDERALLY SPONSORED RESEARCH OR DEVELOPMENT

Not Applicable

REFERENCE TO SEQUENCE LISTING, A TABLE, OR A COMPUTER PROGRAM LISTING COMPACT DISC APPENDIX

Not Applicable

FIELD OF THE INVENTION

The present invention relates generally to energy conversion systems. More specifically, it pertains to the field of systems for conversion of kinetic and thermal energies of gases into usable work. The invention has applications in renewable power generation, waste heat utilisation, and other areas where the conversion of kinetic and thermal energies of gases into mechanical or electrical work, or both, is desired.

DESCRIPTION OF RELATED ART

The reference in this specification to any prior publication (or information derived from it), or to any matter which is known, is not, and should not be taken as an acknowledgment or admission or any form of suggestion, that the prior publication (or information derived from it) or known matter forms part of the common general knowledge in the field of endeavour, to which this specification relates.

Traditional energy conversion systems often involve the use of turbines to convert the kinetic energy of gases into mechanical work. These systems typically include a turbine through which gas is passed and energy is extracted. For example, wind turbines are widely used to convert the kinetic energy of wind into electrical energy. They consist of rotor blades that capture the wind's kinetic energy, which is then converted into mechanical energy to drive a generator.

One significant disadvantage of wind turbines is their inability to capture the thermal energy of the gas flow, which leads to a substantial amount of wasted energy. This limitation reduces the overall efficiency of the system and increases operational costs due to the underutilisation of available energy resources. Given this limitation, there is a need for an improved energy conversion system that maximises the extraction of usable work from both the kinetic and thermal energies of gases.

OBJECT OF THE INVENTION

The object of the present invention is to provide a device for conversion of kinetic and thermal energies of gases to work that overcomes the drawbacks of the prior art. Specifically, the invention aims to:

- Maximise the extraction of usable work from both the kinetic and thermal energies of the gases.

- Provide control over the energy conversion process through real-time monitoring and adjustment of operational parameters.
- Offer a versatile solution that can be adapted to various industrial, waste heat management, and renewable power generation applications.
- Reduce operational costs and environmental impact by improving the overall efficiency of the energy conversion process.

Other objects and advantages of the present invention will become apparent from the following description, taken in connection with the accompanying drawings, wherein, by way of illustration and example, embodiments of the present invention are disclosed.

SUMMARY OF THE INVENTION

According to the present invention, the device for the conversion of kinetic and thermal energies of gases to work, comprises a duct having an inlet for gas flow entry and an outlet for gas flow exit to the ambient atmosphere; a cascade of at least a first turbine and a second turbine positioned within the duct, wherein the first turbine is positioned in a first zone located between the duct inlet and said first turbine, and the second turbine is positioned in a second zone located between the first turbine and the duct outlet; a post-turbine gas cooling system associated with the first turbine, comprising at least one injector of evaporating liquid positioned at or near the outlet of the first turbine, and at least three pressure gauges and at least three temperature gauges distributed in the first zone, the second zone, and outside the duct, and a controller for controlling the injector based on the pressure and temperature data from the gauges.

At least one pressure gauge and at least one temperature gauge are positioned in each of the first and second zones for measuring respective pressures and temperatures and transferring real-time data to the controller. At least one pressure gauge and at least one temperature gauge are positioned outside the duct for measuring the ambient atmospheric pressure and temperature and transferring real-time data to the controller.

The injector is connected to a liquid's line comprising pipes, at least one reservoir for the liquid, at least one filter and at least one pump. The injector injects a mist of evaporating liquid into the gas flow post the first turbine, inducing rapid cooling and a consequent pressure drop at or near the outlet of the first turbine, wherein the controller uses an algorithm configured to regulate the injection rate of the evaporating liquid based on the real-time pressure and temperature data to maintain preferred pressure and temperature differentials between the first and second zones, and between the second zone and the ambient atmosphere.

The first turbine converts a fraction of the kinetic energy of the gas flow entering the duct into work, with its conversion rate enhanced by the pressure and temperature differentials created by the gas flow's cooling system between the turbine's inlet and outlet. The second turbine is used to prevent ambient air intrusion into the second zone and to further convert a fraction of the residual kinetic energy of the gas flow in the second zone to work. The work produced by the device is the cumulative work of the first and second turbines.

The first and the second turbines are coupled either to electric generators or to mechanical devices or both for converting the turbines work output either to electricity or to mechanical work or to both.

BRIEF DESCRIPTION OF THE DRAWING

By way of illustration only, an embodiment of the invention is described more fully hereinafter with reference to the accompanying drawing, in which FIG. **1** shows an embodiment of a device for the conversion of kinetic and thermal energies of gases to work.

DETAILED DESCRIPTION OF THE INVENTION

The embodiment of the device for the conversion of kinetic and thermal energies of gases to work is shown with references to FIG. **1**.

FIG. **1** shows the device for the conversion of kinetic and thermal energies of gases to work, wherein said device comprises at least one cascade of turbines. Said cascade comprises at least of two turbines-Turbine **1** and Turbine **2**, which are positioned inside a Duct **3**. Duct **3** has an inlet for gas flow entry and an outlet for gas flow exit to the ambient atmosphere. Duct **3** is zoned in at least two zones: Zone **1**, which is located between the inlet of Duct **3** and Turbine **1** and Zone **2**, which is located between Turbine **1** and Turbine **2**. Turbine **2** is located at or near the outlet of Duct **3**.

Alternative embodiments of said device may comprise multiple cascades of turbines and more than two turbines in a cascade.

Turbine **1** converts a fraction of the gas flow's kinetic energy into work, with its performance enhanced by the pressure and temperature differentials between Zones **1** and **2** created by a post-turbine gas cooling system. Turbine **2** is used to prevent ambient air intrusion into Zone **2** and to further convert a fraction of the residual kinetic energy of the gas flow in said zone to work.

The work produced by the device is the cumulative work of Turbines **1** and **2**.

Said post-turbine gas cooling system comprises at least one Injector **4** of an evaporating liquid into the gas flow, at least three Pressure Gauges **5**, at least three Temperature Gauges **6**, and a mechanical or a digital Controller **7** that controls the Injector **4**.

Injector **4** is positioned at or near the outlet of Turbine **1** (the pipes and the reservoir and the filter and the pump of the liquid's line are not shown in the FIGURE).

At least one Pressure Gauge **5** and at least one Temperature Gauge **6** are located in Zone **1**, measuring the pressure and the temperature in said zone and transferring the real-time data to the Controller **7**.

At least one Pressure Gauge **5** and at least one Temperature Gauge **6** are located in Zone **2**, measuring the pressure and the temperature in said zone and transferring the real-time data to the Controller **7**.

At least one Pressure Gauge **5** and at least one Temperature Gauge **6** are located outside the Duct **3**, measuring the pressure and the temperature of the ambient atmosphere and transferring the real-time data to the Controller **7**.

Injector(s) **4** inject(s) mist of evaporating liquid into the gas flow. The mist of evaporating liquid mixes with the gas flow and evaporates. The evaporation of the liquid rapidly cools the gas flow, which creates a pressure drop at or near the outlet of Turbine **1**. The pressure difference that is created between Zones **1** and **2** of the Duct **3** results into a partial utilisation of the thermal energy of the gas flow to work.

Controller **7** receives pressure and temperature real-time data from the Pressure Gauges **5** and the Temperature Gauges **6** for Zones **1** and **2** of the Duct **3**, and for the ambient atmosphere. Controller **7** uses an algorithm to optimise the injection rate of the evaporating liquid based on real-time pressure and temperature data. Based on this data, Controller **7** commands Injector(s) **4** to inject an amount of the evaporating liquid at a rate that is sufficient to create pressure and temperature drops between Zones **1** and **2**, and between Zone **2** and the ambient atmosphere, wherein the pressure and the temperature of the gas flow in Zone **1** is the higher than the pressure and the temperature of the gas flow in Zone **2**, and the pressure and the temperature of the gas flow in Zone **2** is higher than the pressure and the temperature of the ambient atmosphere.

Turbines **1** and **2** are coupled either to electric generators or to mechanical devices or both for converting the turbines work output either to electricity or to mechanical work or to both.

The invention claimed is:

1. A device for conversion of kinetic and thermal energies of gas flows to work, comprising:

(a) a duct having an inlet for gas flow entry and an outlet for gas flow exit to ambient atmosphere;

(b) a first turbine positioned within the duct such that a first zone is located between the duct inlet and the first turbine;

(c) a second turbine positioned within the duct such that a second zone is located between the first turbine and the second turbine, the second turbine being arranged downstream of the first turbine and upstream of the duct outlet;

(d) at least one injector positioned in the second zone downstream of the first turbine and upstream of the second turbine, the injector being configured to inject a mist of evaporating liquid into the gas flow after the gas flow has passed through the first turbine;

(e) a liquid line connected to the injector and comprising pipes, at least one reservoir for the liquid, at least one filter, and at least one pump;

(f) a first pair of gauges positioned in the first zone, the first pair comprising a pressure gauge and a temperature gauge for measuring local pressure and temperature in the first zone;

(g) a second pair of gauges positioned in the second zone, the second pair comprising a pressure gauge and a temperature gauge for measuring local pressure and temperature in the second zone;

(h) a third pair of gauges positioned outside the duct, the third pair comprising a pressure gauge and a temperature gauge for measuring ambient atmospheric pressure and temperature; and

(i) a controller configured to receive real-time pressure and temperature data from the first pair, the second pair, and the third pair and to regulate an injection rate of the evaporating liquid through the injector based on said real-time pressure and temperature data;

wherein the injector injects a mist of evaporating liquid into the gas flow in the second zone after passage through the first turbine, evaporation of the liquid rapidly cooling the gas flow and causing a pressure drop from the first zone to the second zone at or near an outlet side of the first turbine, and the controller maintaining pressure and temperature in the second zone above ambient atmospheric pressure and temperature.

2. The device of claim **1**, wherein the second turbine is positioned at or near the duct outlet and is configured to oppose ingress of ambient air from the duct outlet into the second zone and to convert a fraction of residual kinetic energy of the gas flow into work.

3. The device of claim **1**, wherein the controller uses an algorithm configured to command the injector to inject the evaporating liquid at a rate sufficient to maintain the pressure and temperature relationships between the first zone, the second zone, and the ambient atmosphere.

* * * * *

Annexure B

System Performance and Design Optimisation Simulator

US Patent 12,650,086 B1: Device for Conversion of Kinetic and Thermal Energies of Gas Flows to Work

1. Simulator algorithm, dual-unit interface and constrained design optimisation

This Annexure specifies a general two-stage model and also permits defined special operable cases to be selected through the project configuration. Unless a special case is invoked, the general model permits work recovery or an applied load at either turbine, a fixed or optimised Zone 2-to-inlet area ratio, and pressure allocation through the general control and component models. Section 17 defines the outlet-isolation special case, in which the downstream rotor is used primarily to maintain outward flow and discourage atmospheric ingress, receives no electrical-generation credit, the inlet and Zone 2 flow areas are equal, and Zone 2 pressure is solved from the coupled forward-flow compatibility of turbine 1, the evaporating control volume and the outlet-isolation rotor. The special case overrides only the variables, equations, objective terms and outputs expressly identified in section 17; all common thermodynamic, spray, property, conservation, constraint, numerical, logging, verification and validation requirements in this Annexure remain applicable.

1.1 Simulator purposes and execution modes

The implementation shall support five execution modes. The modes use the same physical kernel and differ only in which quantities are prescribed and which are solved or optimised.

Mode	Prescribed quantities	Primary solved quantities	Principal use
Forward performance	Inlet/ambient conditions and a fixed system design	Station states, liquid demand, gross and net power, energy and constraint reserves	Evaluate one existing or proposed design
Inverse sizing	Inlet/ambient conditions and target net power or energy	Required flow scale, inlet area, principal dimensions and equipment ratings	Size a system for a stated output
Design optimisation	Design bounds, scenarios and constraints	Recommended fixed geometry, turbine/spray/pump/generator parameters	Maximise energy across the design envelope
Operating optimisation	A fixed design and current or forecast conditions	Pressure factor, injection, speeds, loads and other adjustable setpoints	Maximise net power without violating constraints
Dynamic and robustness test	A fixed design, controller settings and disturbances	Time histories, trips, settling, energy, margins and uncertainty statistics	Test the recommendation before physical implementation

The complete user input record is represented by:

(1) $$z_{\text{user}} = \left[u_{\text{sys}}, \text{mode}, z_{\text{boundary}}, p_{\text{fixed}}, d_{\text{bounds}}, u_{\text{bounds}}, q_{\text{scen}}, q_{\text{unc}}, q_{\text{solver}}\right]^T$$

where u_{sys} identifies the declared unit system; z_{boundary} contains inlet and atmospheric conditions; p_{fixed} contains known physical constants and calibrated coefficients; d_{bounds} and u_{bounds} define fixed-design and operating-variable bounds; q_{scen} defines deterministic scenarios or time series; q_{unc} defines uncertain distributions; and q_{solver} defines numerical tolerances, seeds and output options.

Every dimensional input is converted to a canonical SI value before any thermodynamic, fluid, structural or optimisation calculation:

(2) $$x_{\text{SI}} = \text{C}\left(x_{\text{user}}, \text{unit}_{\text{user}}, \text{quantitytype}, \text{pressurereference}, \text{temperaturekind}\right)$$

The display layer converts the canonical result to the requested output units without changing the stored SI result:

(3) $$x_{\text{display}} = C^{-1}\left(x_{\text{SI}}, \text{unit}_{\text{display}}, \text{quantitytype}, \text{pressurereference}, \text{temperaturekind}\right)$$

For $\text{s} = 1, \ldots, N_s$ deterministic operating scenarios and $\text{r} = 1, \ldots, N_r$ uncertainty realisations, the simulator evaluates:

(4) $$R_{\text{s,r}} = \text{Simulate}\left(d, u_{\text{s,r}}, z_{\text{s,r}}, p_{\text{fixed}}\right)$$

The persistent result object shall contain the values, units, feasibility state, active constraints, solver diagnostics, warnings, source inputs and reproducibility metadata:

$$R = \{\text{values}_{\text{SI}}, \text{values}_{\text{display}}, \text{units}, \text{feasible}, \text{constraints}, \text{diagnostics}, \text{warnings}, \text{provenance}\} \quad (5)$$

Simulator execution and recommendation workflow

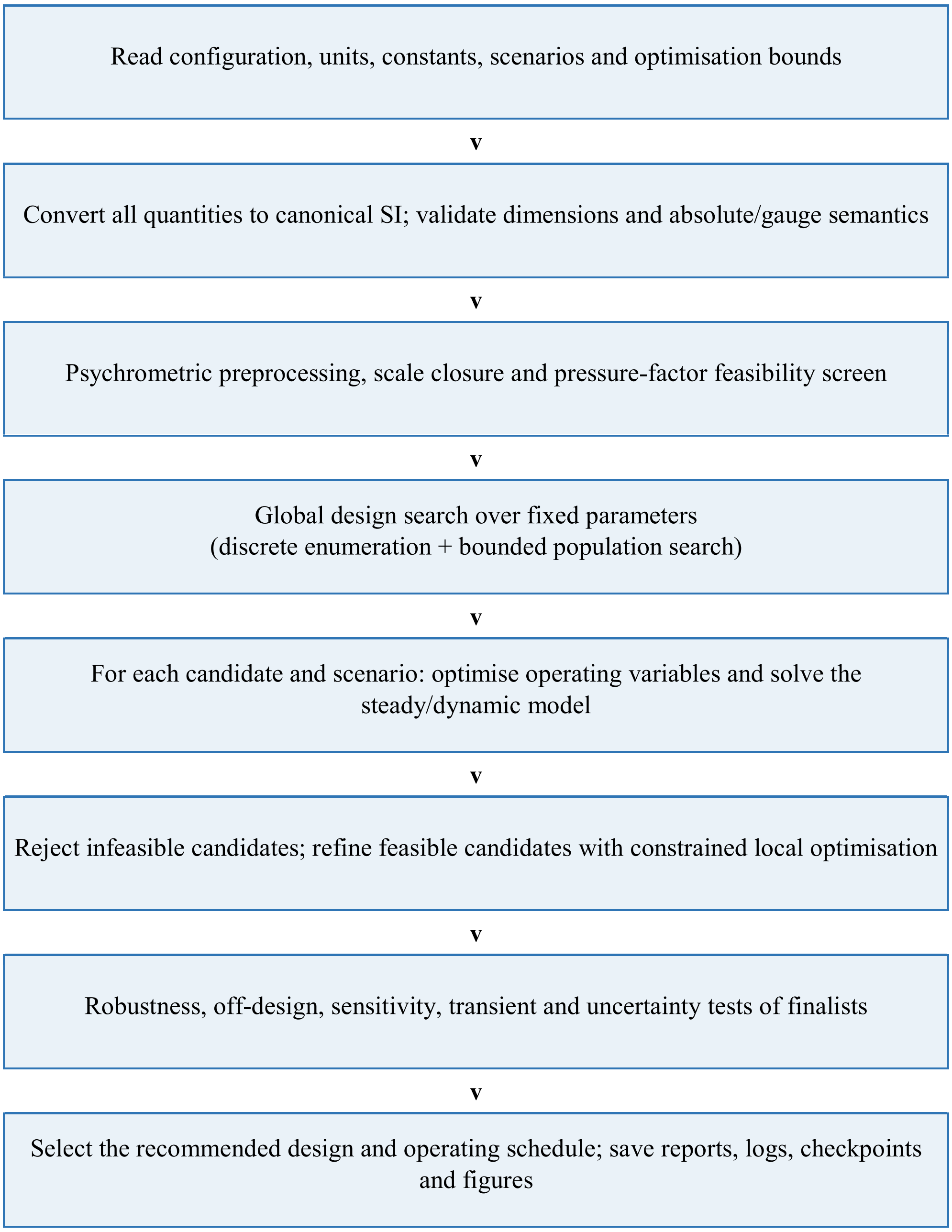

Figure 1.1. Simulator execution and recommendation workflow.

1.2 Separation of fixed design and operational variables

The optimisation must not treat all variables as interchangeable. Fixed design variables are selected once for construction; operational variables can change during operation; state variables are solved by the physical model; uncertain variables are sampled or supplied as scenarios.

Class	Examples	Treatment
Fixed design d	A_1, A_2/A_1 , duct diameters and lengths, rotor and hub diameters, blade counts or map identifiers, Zone 2 volume, nozzle count and orifice area, liquid-line diameter, reservoir size, pump and generator ratings	Optimised in the outer design loop; fixed during each operational simulation
Operational u	F_P , liquid mass flow, valve/pump command, N_1 and N_2 or torque/load setpoints, variable pitch/stator settings, drain fraction and bypass command where available	Optimised separately for each scenario or time interval, subject to rate, map and safety limits
State x	P_{1e}, T_{1e}, P_2, T_2 , humidity ratio, density, velocity, P_3, T_3 , stored masses, rotor speeds and controller states	Solved from the residual equations or dynamic differential-algebraic equations
Uncertain ξ	Inlet velocity, pressure, temperature, humidity, losses, turbine efficiencies, evaporation coefficient, fouling, sensor bias and auxiliary demand	Sampled using deterministic envelopes, Latin-hypercube/Monte Carlo realisations or supplied probability distributions

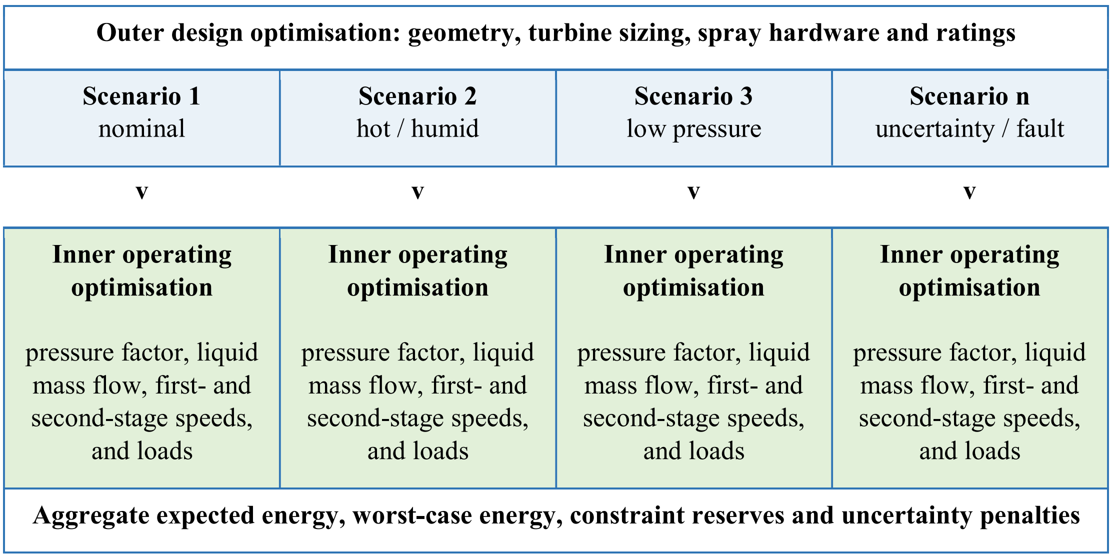

Figure 1.2. Nested design and operating optimisation.

1.3 Fidelity levels

Level	Physical content	Use and restriction
L0 - screening	Algebraic steady-state model, constant efficiencies, prescribed pressure losses and simple evaporation correlation	Fast rejection and coarse global search only; not a final recommendation
L1 - engineering	Full residual system, psychrometrics, D^2 evaporation law, calculated pressure losses, shaft/electrical sizing and dynamic controller model	Default recommendation level when calibrated maps are not available
L2 - calibrated	Measured turbine maps, pump/valve curves, nozzle map, evaporation calibration, heat loss and measured auxiliary demands	Preferred for prototype prediction and operating optimisation
L3 - co-simulation	L2 model coupled to CFD, finite-element or external component solvers at selected candidate points	Finalist verification; too expensive for every global-search point

2. Unit-system architecture and conversion requirements

All calculations shall be performed internally in coherent SI units: metre, kilogram, second, kelvin, pascal, joule, watt, mole and derived SI units. US customary values are accepted and displayed through a unit registry. Each input field must carry both a numeric value and an explicit unit token. Bare dimensional numbers are rejected unless a configuration-level default is unambiguous.

2.1 Temperature conversions

Absolute temperatures and temperature differences must be treated as different quantity types. An offset is applied to absolute temperatures but never to a temperature difference.

(6a) $$T_{\mathrm{K}} = \theta_{^\circ\mathrm{C}} + 273.15$$

(6b) $$T_{\mathrm{K}} = \left(\theta_{^\circ\mathrm{F}} + 459.67\right)\frac{5}{9}$$

(6c) $$T_{^\circ\mathrm{R}} = T_{\mathrm{K}}\frac{9}{5} \text{ or equivalently } T_{\mathrm{K}} = \mathrm{T}_{^\circ\mathrm{R}}\frac{5}{9}$$

(6d) $$\Delta T_{\mathrm{K}} = \Delta T_{^\circ\mathrm{C}} = \Delta T_{^\circ\mathrm{F}}\frac{5}{9} = \Delta T_{^\circ\mathrm{R}}\frac{5}{9}$$

2.2 Pressure conversions and reference semantics

The simulator shall distinguish absolute pressure, gauge pressure and differential pressure. Thermodynamic equations, density calculations, saturation calculations and pressure ratios shall use absolute pressure. Gauge inputs are converted using the atmospheric pressure associated with the same scenario.

(7) $$P_{\mathrm{abs}} = P_{\mathrm{gauge}} + P_{\mathrm{atm}}$$

(8a) $$P_{\mathrm{Pa}} = 1000P_{\mathrm{kPa}} = 10^5 P_{\mathrm{bar}} = 6894.757293P_{\mathrm{psi}}$$

(8b) $$P_{\mathrm{Pa}} = 3386.389P_{\mathrm{inHg}} = 249.08891P_{\mathrm{inH_2O}}$$

The parser shall require pressure unit tokens such as psia, psig, kPa(a), kPa(g), Pa(d) or psi(d). A value labelled only psi or kPa is rejected in strict mode and accepted only with an explicit project default in permissive mode.

2.3 Flow, geometry, power and property conversions

(9) $U_{\text{m/s}} = 0.3048 U_{\text{ft}/s} = 0.44704 U_{\text{mph}}$

(10) $\dot{m}_{\text{kg/s}} = 0.45359237\ \dot{m}_{\text{lbm/s}}$

(11a) $\dot{V}_{\text{m}^3/\text{s}} = 0.028316846592\ \dot{V}_{\text{ft}^3/\text{s}}$

(11b) $\dot{V}_{\text{m}^3/\text{s}} = 6.30901964 \times 10^{\ 5} \dot{V}_{\text{USgpm}}$

(12a) $L_{\text{m}} = 0.3048 L_{\text{ft}} = 0.0254 L_{\text{in}}$

(12b) $A_{\text{m}^2} = 0.09290304 A_{\text{ft}^2},\ \ V_{m^3} = 0.028316846592 V_{\text{ft}^3}$

(13) $\dot{W}_{\text{W}} = 745.699872\ \dot{W}_{\text{hp}}$

(14) $E_{\text{J}} = 1055.055853 E_{\text{Btu}}, E_{\text{kWh}} = \dfrac{E_{\text{Btu}}}{3412.141633}$

(15) $\tau_{\text{Nm}} = 1.355817948 \tau_{\text{lbfft}}, F_{\text{N}} = 4.448221615 F_{\text{lbf}}$

(16) $\rho_{\text{kg/m}^3} = 16.01846337 \rho_{\text{lbm}/\text{ft}^3}$

(17a) $\mu_{\text{Pas}} = 10^{-3} \mu_{\text{cP}} = 1.488163944 \mu_{\text{lbm/(fts)}}$

(17b) $\nu_{\text{m}^2/\text{s}} = 0.09290304 \nu_{\text{ft}^2/\text{s}}$

(18) $k_{\text{W/(mK)}} = 1.730734666 k_{\text{Btu/(hft}\cdot\text{F)}}$

(19a) $c_{\text{p,J/(kg}\cdot\text{K)}} = 4186.800584 c_{\text{p,Btu/(lbm}\cdot\text{F)}}$

(19b) $h_{\text{J/kg}} = 2326.000324 h_{\text{Btu/lbm}}$

(20) $\sigma_{\text{N/m}} = 14.59390294 \sigma_{\text{lbf/ft}}$

(21) $\Omega_{\text{rad/s}} = \dfrac{2\pi\pi_{\text{rpm}}}{60}$

(22) $\phi_{\text{fraction}} = \dfrac{\phi_{\text{percent}}}{100}, \eta_{\text{fraction}} = \dfrac{\eta_{\text{percent}}}{100}$

Quantity	**Canonical SI**	**Accepted SI examples**	**Accepted US customary examples**
Pressure	Pa absolute	Pa(a), kPa(a), bar(a); gauge/differential with qualifier	psia, psig, psi(d), inHg(a), $\text{inH}_2\text{O(d)}$
Temperature	K absolute	K, °C; K-difference, °C-difference	°F, °R; °F-difference, °R-difference
Velocity	m/s	m/s, km/h	ft/s, mph
Mass flow	kg/s	kg/s, kg/h	lbm/s, lbm/min, lbm/h

Volumetric flow	m^3/s	m^3/s, m^3/h, L/s	ft^3/s, cfm, US gpm for liquid
Length/area/volume	m, m^2, m^3	mm, cm, m; L	in, ft; in^2, ft^2; US gal, ft^3
Power/energy	W, J	W, kW, MW; J, MJ, kWh	hp, Btu/h; Btu, MMBtu, hp·h
Torque/force	N·m, N	N·m, kN	lbf·ft, lbf
Density/viscosity	kg/m^3, Pa·s, m^2/s	kg/m^3, cP, cSt	lbm/ft^3, lbm/(ft·s), ft^2/s
Heat properties	J/(kg·K), J/kg, W/(m·K)	kJ/(kg·K), kJ/kg	Btu/(lbm·°F), Btu/lbm, Btu/(h·ft·°F)
Vibration	m/s or m/s^2	mm/s, m/s^2	in/s, g
Humidity/efficiency	dimensionless	fraction or %	fraction or %

The unit registry shall also store significant-digit preferences, allowed aliases, dimensionality, whether offset conversion is permitted, and whether the quantity is absolute, gauge, differential or dimensionless. Conversion tests must include round-trip checks in both directions.

3. Input, parameter and output data model

3.1 Parameter records

The fixed parameter record shall be versioned. Every calibrated coefficient shall include a source, date, validity range and uncertainty. Default values may be used for exploratory simulation but shall be flagged as assumed rather than measured.

(23) $$p_{\text{fixed}} = \{p_{\text{fluid}}, p_{\text{loss}}, p_{\text{turbine}}, p_{\text{spray}}, p_{\text{pump}}, p_{\text{generator}}, p_{\text{structure}}, p_{\text{control}}\}$$

The fixed-design decision vector may be expanded or contracted according to available component models. A recommended minimum vector is:

(24) $$d = [A_1, r_A, L_{Z2}, D_{\text{tip},1}, \lambda_{\text{h},1}, D_{\text{tip},2}, \lambda_{\text{h},2}, n_{\text{or}}, A_{\text{or,total}}, D_l, V_{\text{res}}, \text{ratings}]^T$$

The operational decision vector for a fixed design is:

(25) $$u = [F_P, \dot{m}_l, N_{1,\text{set}}, N_{2,\text{set}}, \tau_{\text{g},1,\text{set}}, \tau_{\text{g},2,\text{set}}, x_{\text{pitch},1}, x_{\text{pitch},2}, f_{\text{drain}}]^T$$

The uncertain-variable vector is:

(26) $$\xi = [U_1, P_1, T_1, \phi_1, P_a, T_a, \phi_a, \Delta P_{\text{loss}}, \eta_j, K_{\text{evap}}, P_{\text{aux}}, \text{sensorbias}]^T$$

The principal output vector is:

(27) $$y_{\text{out}} = [P_i, T_i, \omega_i, \rho_i, U_i, \dot{m}_i, \dot{W}_{\text{sh,j}}, \dot{W}_{\text{el,j}}, \dot{W}_p, \dot{W}_{\text{net}}, E_{\text{net}}, \text{dimensions}, \text{reserves}]^T$$

Each text-log record shall contain an ISO-8601 timestamp or simulation time, run identifier, scenario identifier, iteration, state, objective, maximum violation and a free-text message:

(28) $$\ell_k = [t_k, \text{run\_id}, \text{scenario\_id}, \text{stage}, \text{iteration}, \text{objective}, \text{g}_{\max}, \text{status}, \text{message}]$$

Group	Input	SI unit	US unit	Meaning	Status
Run control	unit_system	-	-	SI or US_CUSTOMARY	Required
Run control	mode	-	-	FORWARD, INVERSE, DESIGN_OPT, OPERATING_OPT, DYNAMIC_TEST	Required
Boundary	U_1	m/s	ft/s or mph	Mean incoming axial velocity	Required
Boundary	P_1	Pa(a)	psia	Zone 1 absolute static pressure	Required
Boundary	T_1	K or °C	°F or °R	Zone 1 absolute temperature	Required
Boundary	ϕ_1	fraction or %	fraction or %	Inlet relative humidity	Required for humid air
Boundary	P_a	Pa(a)	psia or inHg(a)	Atmospheric absolute pressure	Required
Boundary	T_a	K or °C	°F or °R	Atmospheric temperature	Required
Boundary	ϕ_a	fraction or %	fraction or %	Ambient relative humidity	Recommended
Scale	A_1 or D_1	m^2 or m	ft^2 or ft	Forward scale closure	One of scale inputs
Scale	$\dot{W}_{\text{net,target}}$ or E_{target}	W or J/kWh	hp or Btu/h ; Btu/kWh	Inverse scale closure	One of scale inputs
Operation	t_{op}	s or h	s or h	Operating duration	Required for energy
Pressure control	F_P	-	-	Normal Zone 2 pressure factor	Bounded/optimised
Pressure control	F_{trip}	-	-	Protective factor, $1 < F_{\text{trip}} < F_P$	Required for dynamic test
Losses	ΔP_{exit}, ΔP_{mix} ΔP_{duct}	Pa	psi or inH_2O	Outlet, mixing and duct losses	Required or calculated
Geometry	$r_A = A_2/A_1$	-	-	Zone 2 to inlet area ratio	Fixed or optimised
Turbines	$\eta_{\text{is,j}}$, $\eta_{\text{m,j}}$ $\eta_{\text{g,j}}$	fraction or %	fraction or %	Stage efficiencies	Required or map-derived
Turbines	map_j	file/reference	file/reference	Corrected flow-speed-pressure map	Preferred
Turbines	$N_{\text{j,max}}$ $U_{\text{tip,j,max}}$	rpm, m/s	rpm, ft/s	Speed and tip-speed constraints	Required

Spray	$T_{\ell,in}, P_{\ell,in}, h_{\ell,in}$, $\rho_\ell, \mu_\ell, c_{p,\ell}, k_\ell, \sigma_\ell$, $P_{crit,\ell}, T_{crit,\ell}$	SI property units	US property units	Liquid supply state, transport properties and critical constants	Required
Spray	$T_{sat}(P), h_{fg}(P)$, property method/version and validity domain	function/reference	function/reference	Pressure-dependent saturation, enthalpy and latent-heat model	Required and versioned
Spray	$D_{32}, K_{evap}, K_{eff}$ correlation and enhancement bounds	m, $\mathrm{m^2/s}$, –	in, $\mathrm{ft^2/s}$, –	Conventional and high-temperature droplet-evaporation model	Required or calibrated
Spray	flash option, iteration tolerance, relaxation factor, maximum iterations	–, –, count	–, –, count	Flash treatment and coupled energy–mass solver settings	Required
Spray	C_d, n_{or} ΔP_{nozzle}	-, count, Pa	-, count, psi	Atomiser parameters	Required/optimised
Spray	$\chi_{evap,min}$ $f_{carry,max}$	fraction or %	fraction or %	Evaporation and carryover limits	Required
Pump	η_p $\eta_{motor,p}$	fraction or %	fraction or %	Pump and motor efficiency	Required
Pump	NPSH_a NPSH_r	m	ft	Cavitation constraint	Recommended
Electrical	P_{aux}, V_L, PF	W, V, -	hp/kW, V, -	Auxiliary demand and generator data	Required for net output
Structure	σ_{allow}, SF, material limits	Pa, -	psi, -	Preliminary stress constraints	Required for sizing
Dynamic	J_j, actuator τ, controller gains	$\mathrm{kg \cdot m^2}$, s	$\mathrm{lbm \cdot ft^2}$, s	Rotor and actuator/control model	Dynamic mode
Optimiser	bounds, tolerances, seed	mixed	mixed	Reproducibility and search settings	Required for optimisation

3.2 Required output groups

Output group	Mandatory contents
Input echo	All entered values, original units, SI values and conversion status
Feasibility	Kernel status, pass/fail, rejected condition, maximum violation, property-domain status, active evaporation regime, active limiting constraint and all constraint reserves
Station states	Pressure, temperature, humidity ratio, density, velocity, Mach and Reynolds number at each station
Mass balance	Dry air, inlet vapour, net flash vapour, gas-driven vapour, excess flash mass

Output group	Mandatory contents
	returned to the liquid inventory, unevaporated liquid, carryover, drain and total mixture flows
Turbines	Pressure ratios, corrected map coordinates, efficiency, speed, torque, shaft power and electrical output
Spray zone	Liquid rate; local saturation temperature and latent heat; gas and liquid superheat; flash fraction and net flash-vapour flow; gas-driven evaporation; total evaporated flow and evaporation fraction; effective evaporation coefficient; representative droplet temperature; finite-capacity or superheated-state status; residence and evaporation times; carryover; heat absorbed; active evaporation regime; active limiting constraint; property method and property-validity status
Parasitics	Pump, controller, sensor, lubrication and miscellaneous power
Energy	Gross power, net power, net energy, specific work and energy-balance residual
Dimensions	Duct areas/diameters/lengths, turbine annuli/rotors/hubs/shafts, nozzle areas, pipe size and reservoir volume
Ratings	Pump, generator, drive, current, apparent power and safety-factor ratings
Optimisation	Candidate rank, objective, convergence, accepted operating variables and active bounds
Robustness	Expected/worst-case energy, probability of feasibility, percentiles, sensitivity and failure modes
Dynamic test	Time histories, settling, overshoot, trips, safe-state proof and accumulated energy
Files	Paths to text logs, restart checkpoint, figures and machine-readable result files

4. Preprocessing, validation and initialisation algorithm

4.1 Input parsing and dimensional validation

1. Load the project configuration and schema version. Refuse unsupported future schema versions and migrate known older versions deterministically.
2. Parse each dimensional field as value plus unit. Convert it to SI and preserve the original value and unit in the input echo.
3. Check dimensional consistency. For example, reject a torque unit in a power field and reject a gauge pressure where an atmospheric absolute pressure is required.
4. Check finite numeric values, lower and upper physical bounds, mandatory fields and mutually exclusive scale closures.
5. Check that every optimisation bound has $\text{lower} < \text{initial} < \text{upper}$, and that discrete choices are valid members of their declared sets.
6. Write input_echo.txt before solving so failed runs still preserve the received data.

(29) $$I_{\text{valid}} = \prod_i 1[\text{finite}(x_i)] 1[\text{dimension}(x_i) = \text{expected}_i] 1[x_{\text{i,min}} \le x_i \le x_{\text{i,max}}]$$

4.2 Thermodynamic preprocessing and feasibility projection

The psychrometric module shall calculate saturation pressure, humidity ratio, dry-air and vapour mass fractions, mixture enthalpy and density on a consistent basis. The pressure-allocation module shall calculate the inlet, interstage and outlet pressure relationships, and the scale module shall apply either the forward area closure or the inverse power/energy closure. The following additional steps make preprocessing robust for optimisation.[47][48] The liquid-property module shall

[47] ASHRAE (2025) ASHRAE Handbook - Fundamentals. Atlanta: American Society of Heating, *Refrigerating and Air-Conditioning Engineers*, Chapter 1, Psychrometrics.

calculate $T_{sat,2} = T_{sat}(P_2)$, $h_{fg,2} = h_{fg}(P_2)$, $h_{t,in}$, critical-state checks and property-domain status from the selected validated formulation. Property calls outside the declared domain, including $P_2 \geq P_{crit,t}$, shall return PROPERTY_ERROR and shall not be extrapolated silently.

(30) $$F_{\mathrm{P,min}} = 1 + \frac{\Delta P_{\mathrm{exit}} + \Delta P_{\mathrm{T2,min}} + \Delta P_{\mathrm{reserve}}}{P_a}$$

(31) $$F_{\mathrm{P,max}} = \frac{P_1 - \Delta P_{\mathrm{mix}} - \Delta P_{\mathrm{12,duct}} - \Delta P_{\mathrm{T1,min}}}{P_a}$$

(32) $$F_{\mathrm{P,0}} = \mathrm{clip}\left(F_{\mathrm{P,user}}, F_{\mathrm{P,min}} + \varepsilon_F, F_{\mathrm{P,max}} - \varepsilon_F\right)$$

(33) $$A_{1,0} = \frac{\dot{m}_{\mathrm{da,guess}}}{C_m \rho_{\mathrm{da,1}} U_1}$$

(34) $$T_{\mathrm{1e,0}} = T_1 - \eta_{\mathrm{is,1}}\left[T_1 - T_1\left(\frac{P_{\mathrm{1e}}}{P_1}\right)^{(\gamma_1\text{-}1)/\gamma_1}\right]$$

(35) $$T_{2,0} = \max\left(T_a + \Delta T_{\mathrm{min}}, T_{\mathrm{1e,0}} - \Delta T_{\mathrm{evap,guess}}\right)$$

(36) $$\dot{m}_{\mathrm{evap,0}} = \mathrm{clip}\left(\frac{\dot{m}_1 c_{\mathrm{p,m,1}}\left(T_{\mathrm{1e,0}} - T_{2,0}\right)}{\Delta h_{\mathrm{evap}}}, 0, \dot{m}_{\mathrm{evap,max}}\right)$$

The full residual system is initialised by continuation. The solver first evaluates zero spray, then ramps the requested spray or pressure factor over N_c continuation steps. This is more reliable than requesting the final highly coupled point in one step.

(37) $$\lambda_{\mathrm{c,k}} = \frac{k}{N_c}, \dot{m}_{\mathrm{1,k}} = \lambda_{\mathrm{c,k}} \dot{m}_{\mathrm{1,target}}, \mathrm{k} = 0, \ldots, N_c$$

(38) $$r_F = \left\| S_F^{-1} F(y; z, p) \right\|_2$$

(39) $$\text{converged} \Leftrightarrow r_{\mathrm{F}} \leq \varepsilon_{\mathrm{F,res}} \wedge \left\| \frac{\Delta y}{S_{\mathrm{y}}} \right\|_\infty \leq \varepsilon_{\mathrm{y}} \wedge g_{\mathrm{max}} \leq \varepsilon_{\mathrm{g}}$$

Engineering limitation: A nonlinear solver convergence flag alone is not a feasible design. The post-solve constraint evaluation is mandatory, including saturation, carryover, turbine-map, Mach, structural, pressure, generator, pump and positive-net-power checks.

5. Steady-state simulation kernel

The steady kernel is the principal model evaluation used by the global search, local design optimisation and operating optimisation. It shall expose both a simultaneous residual formulation and a sequential fallback. The simultaneous form is preferred near a well-scaled solution; the sequential form is useful for initialisation and diagnostics.

5.1 Inlet flow and pressure allocation

(40) $$\dot{m}_{\mathrm{da}} = C_m \rho_{\mathrm{da,1}} A_1 U_1, \dot{m}_{\mathrm{v,1}} = \omega_1 \dot{m}_{\mathrm{da}}, \dot{m}_1 = (1 + \omega_1)\dot{m}_{\mathrm{da}}$$

(41) $$P_2 = F_P P_a, P_3 = P_a + \Delta P_{\mathrm{exit}}, P_{\mathrm{1e}} = P_2 + \Delta P_{\mathrm{mix}} + \Delta P_{\mathrm{12,duct}}$$

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[48] Bell, I.H., Lemmon, E.W. and Harvey, A.H. (2017) Algorithms for the calculation of psychrometric properties from multi-fluid Helmholtz-energy-explicit models, *International Journal of Refrigeration*, 84, pp. 59-70; International Association for the Properties of Water and Steam (IAPWS) (2012) *Revised Release on the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam*, R7-97(2012). For a general or scientific water-property formulation, use IAPWS R6-95(2018) within its declared domain.

5.2 First turbine

(42a) $$T_{1\mathrm{e,s}} = T_1\left(\frac{P_{1\mathrm{e}}}{P_1}\right)^{(\gamma_1\text{-}1)/\gamma_1}$$

(42b) $$T_{1\mathrm{e}} = T_1 - \eta_{\mathrm{is},1}\left(T_1 - T_{1\mathrm{e,s}}\right)$$

(43a) $$\dot{W}_{\mathrm{sh},1} = \eta_{\mathrm{m},1}\dot{m}_1\left[c_{\mathrm{p,m},1}\left(T_1 - T_{1\mathrm{e}}\right) + \frac{U_1^2 - U_{1\mathrm{e}}^2}{2}\right]$$

(43b) $$\dot{W}_{\mathrm{el},1} = \eta_{\mathrm{g},1}\dot{W}_{\mathrm{sh},1}$$

When a validated turbine map exists, equations (42)-(43) are replaced by interpolation in corrected mass flow, corrected speed and pressure ratio, with extrapolation prohibited.[49]

5.3 Spray-zone coupled solution

(44) $$\omega_2 = \omega_1 + \frac{\dot{m}_{\mathrm{evap}}}{\dot{m}_{\mathrm{da}}}, \dot{m}_2 = \dot{m}_1 + \dot{m}_{\mathrm{evap}}$$

(45) $$\dot{m}_{\mathrm{evap,max}} = \dot{m}_{\mathrm{da}}\max\left[\omega_{\mathrm{sat}}\left(T_2, P_2\right) - \omega_1, 0\right]$$

(46) $$E_{\mathrm{spray}} = \dot{m}_{\mathrm{da}}h_{\mathrm{ma}}\left(T_{1\mathrm{e}}, \omega_1\right) + \dot{m}_l h_l\left(T_l\right) - \dot{m}_{\mathrm{da}}h_{\mathrm{ma}}\left(T_2, \omega_2\right) - \left(\dot{m}_l - \dot{m}_{\mathrm{evap}}\right)h_l\left(T_2\right) - \dot{Q}_{\mathrm{loss},2}$$

(47) $$\chi_{\mathrm{evap}} = 1 - \left\{\max\left[1 - \frac{K_{\mathrm{evap}}t_{\mathrm{res},2}}{D_{32}^2}, 0\right]\right\}^{3/2}$$

(48) $$\dot{m}_l = \frac{\dot{m}_{\mathrm{evap}}}{\max\left(\chi_{\mathrm{evap}}, \chi_{\mathrm{floor}}\right)}, \dot{m}_{\mathrm{carry}} = \max\left[\left(1 - \chi_{\mathrm{evap}}\right)\dot{m}_l - \dot{m}_{\mathrm{drain}}, 0\right]$$

(49a) $$\rho_2 = \frac{P_2}{R_{\mathrm{m},2}T_2}, U_2 = \frac{\dot{m}_2}{\rho_2 A_2}, t_{\mathrm{res},2} = \frac{L_{\mathrm{Z2}}}{U_2}$$

The unknowns T_2, $\dot{m}_{\mathrm{evap}}$, $\dot{m}_l$ and U_2 are obtained by simultaneously satisfying the energy balance, saturation constraint, evaporation-kinetics relationship, continuity and the one-dimensional momentum residual implemented by the steady physical kernel. If the saturation boundary is active, the solver shall use a complementarity or active-set treatment rather than allowing supersaturation silently.[50]

5.3.1 Regime-aware boiling, flash and high-temperature submodel

Equations (44)–(49a) remain the base spray-zone system. Before each coupled spray iteration, the steady kernel shall calculate the pressure-dependent saturation state and select the applicable conventional, high-temperature, boiling-limited or flash-assisted branch.

(49b) $$P_{sat}\left(T_{sat,2}\right) = P_2, P_2 < P_c$$

(49c) $$h_{fg,2} = h_{fg,r}\left[\frac{1 - T_{sat,2}/T_c}{1 - T_r/T_c}\right]^{n_W}$$

(49d) $$x_{flash,c} = \mathrm{clip}\left[\frac{h_\ell\left(T_{\ell,in}\right) - h_f\left(P_2\right)}{h_{fg,2}}, 0, 1\right)$$

(49e) $$\dot{m}_{flash,c} = x_{flash,c}\dot{m}_\ell, \dot{m}_{\ell,r,c} = \dot{m}_\ell - \dot{m}_{flash,c}$$

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[49] Dixon, S.L. and Hall, C.A. (2014) *Fluid Mechanics and Thermodynamics of Turbomachinery*. 7th edn. Oxford: Butterworth-Heinemann.
[50] Sirignano, W.A. (2010) *Fluid Dynamics and Transport of Droplets and Sprays*. 2nd edn. Cambridge: Cambridge University Press.

(49f) $$F_{HT} = \min\left(F_{HT,max}, 1 + \frac{\max\left(T_{1e} - T_{sat,2}, 0\right)}{\Delta T_{HT,ref}} \right), K_{eff} = K_{evap} F_{HT}$$

(49g) $$\chi_{kin} = 1 - \left[\max\left(1 - \frac{K_{eff} t_{res,2}}{D_{32}^2}, 0 \right) \right]^{3/2}$$

At each inner iteration, the saturation-capacity calculation shall use the current T_2. When $P_{sat}(T_2)$ is below P_2, the net flash vapour and subsequent gas-driven evaporation are constrained by the finite vapour capacity. When $P_{sat}(T_2)$ is equal to or greater than P_2, the gas is superheated relative to liquid–vapour equilibrium and no artificial finite humidity cap is imposed:

(49h) $$\dot{m}_{cap} = \begin{cases} \dot{m}_{da} max\left[\omega_{sat}(T_2, P_2) - \omega_1, 0\right), & P_{sat}(T_2) < P_2, \\ +\infty, & P_{sat}(T_2) \geq P_2. \end{cases}$$

(49i) $$\dot{m}_{flash} = \min\left(\dot{m}_{flash,c}, \dot{m}_{cap}\right), \dot{m}_{\ell,r} = \dot{m}_{\ell} - \dot{m}_{flash}$$

(49j) $$T_{\ell,r} = \begin{cases} T_{sat,2}, & \dot{m}_{flash} > 0, \\ T_{\ell,in}, & \dot{m}_{flash} = 0, \end{cases} T_d = \max\left[T_{\ell,r}, \min\left(T_2, T_{sat,2}\right)\right)$$

(49k) $$\dot{Q}_{sens} = \dot{m}_{\ell,r} c_{p\ell} \max\left(T_d - T_{\ell,r}, 0\right)$$

(49l) $$\dot{m}_{evap,E} = \max\left[\frac{\dot{C}_g\left(T_{1e} - T_{2,min}\right) - \dot{Q}_{sens}}{h_{fg,2}}, 0 \right), \dot{C}_g = \dot{m}_{da}\left(c_{pa} + \omega_1 c_{pv}\right)$$

(49m) $$\dot{m}_{evap,g} = \min\left[\chi_{kin} \dot{m}_{\ell,r}, \dot{m}_{evap,E}, \max\left(\dot{m}_{cap} - \dot{m}_{flash}, 0\right)\right)$$

(49n) $$\dot{m}_{evap} = \dot{m}_{flash} + \dot{m}_{evap,g}, \dot{C}_g\left(T_{1e} - T_2\right) - \dot{Q}_{sens} - \dot{m}_{evap,g} h_{fg,2} - \dot{Q}_{loss,2} = 0$$

(49o) $$R \in \{\text{conventional}, \text{high - temperature sensible - heating}, \text{boiling - limited}, \text{flash - assisted})$$

The implementation shall solve T_2, ω_2, density, velocity, residence time, net flash vapour and gas-driven evaporation by relaxed fixed-point iteration, Newton iteration or an equivalent bounded residual method. Flash vapour that exceeds a finite final-state vapour capacity shall be returned to the liquid inventory rather than being retained as impossible supersaturation. The solver shall not replace P_{sat} by an arbitrary fraction of P_2.

The kernel return structure shall add $T_{sat,2}$, $h_{fg,2}$, gas superheat, liquid superheat, flash fraction, flash mass flow, gas-driven evaporation, effective K_{eff}, representative droplet temperature, active regime and active evaporation limit.

PROPERTY_ERROR shall be returned for P_2 at or above the critical pressure, invalid property values or liquid temperature outside the declared property domain. CONVERGED_INFEASIBLE shall be returned when minimum evaporation, carryover, temperature, pressure or Mach constraints fail.

The no-spray baseline shall set both flash and gas-driven evaporation to zero. Verification shall include conventional sub-boiling cases, gas temperatures above $T_{sat,2}$, liquid temperatures above $T_{sat,2}$, transition cases on both sides of the boiling boundary, finite-capacity re-entry after cooling, zero spray and property-domain rejection.

5.4 Second turbine and parasitic demand

(50) $$T_{3,s} = T_2\left(\frac{P_3}{P_2}\right)^{(\gamma_2\text{-}1)/\gamma_2}, T_3 = T_2 - \eta_{is,2}\left(T_2 - T_{3,s}\right)$$

(51) $$\dot{W}_{\mathrm{el},2} = \eta_{\mathrm{g},2}\eta_{\mathrm{m},2}\dot{m}_2\left[c_{\mathrm{p,m},2}\left(T_2 - T_3\right) + \frac{U_2^2 - U_3^2}{2}\right]$$

(52) $$\dot{W}_p = \frac{\dot{m}_l \Delta P_p}{\rho_l \eta_p \eta_{\mathrm{motor,p}}}$$

(53) $$\dot{W}_{\mathrm{net}} = \dot{W}_{\mathrm{el},1} + \dot{W}_{\mathrm{el},2} - \dot{W}_p - \dot{W}_{\mathrm{aux}}$$

(54) $$E_{\mathrm{net}} = \int_0^{t_{\mathrm{op}}} \dot{W}_{\mathrm{net}}(t)\mathrm{d}t, \mathrm{E}_{\mathrm{net}} = \dot{W}_{\mathrm{net}} t_{\mathrm{op}} \text{for a steady case}$$

(55) $$\eta_{\mathrm{net,available}} = \frac{\dot{W}_{\mathrm{net}}}{\dot{m}_1\left(h_{0,1} - h_{\mathrm{ref}}\right)}$$

(56) $$G_{\mathrm{evap}} = \dot{W}_{\mathrm{net,withspray}} - \dot{W}_{\mathrm{net,nospray}}$$

The no-spray baseline shall retain the same geometry and source conditions and shall re-optimise only the operational variables that remain available without injection. This avoids attributing unrelated design changes to evaporative conditioning.

5.5 Residual, constraint and status return

(57) $$F_{\mathrm{sim}} = \left[F_1, \ldots, F_{15}, E_{\mathrm{spray}}, M_{\mathrm{spray}}, F_{\mathrm{map},1}, F_{\mathrm{map},2}\right]^T$$

(58) $$g_j\left(y, d, u, z, p\right), \text{feasible} \Leftrightarrow g_{\max} \leq \varepsilon_g \wedge \dot{W}_{\mathrm{net}} > 0$$

The kernel shall never return only a numeric power. It must return a structured status: CONVERGED_FEASIBLE, CONVERGED_INFEASIBLE, NONCONVERGED, PROPERTY_ERROR, MAP_OUTSIDE_DOMAIN, UNIT_ERROR, INPUT_ERROR or NUMERICAL_FAILURE. The optimiser may rank only CONVERGED_FEASIBLE candidates as valid recommendations. PROPERTY_ERROR shall be returned when the Zone 2 pressure is at or above the liquid critical pressure, when a saturation, enthalpy or latent-heat property call returns an invalid value, when the supplied-liquid temperature lies outside the declared property domain, or when evaluation would require silent extrapolation outside the selected property formulation. CONVERGED_INFEASIBLE shall be returned when the coupled state converges numerically but one or more minimum-evaporation, carryover, Zone 2 temperature, pressure-reserve, turbine-map, Mach-number or other hard constraints fail. NONCONVERGED shall be returned when the bounded energy–mass–property iteration and all configured recovery attempts fail to converge.

6. Dynamic simulation kernel and event handling

Dynamic mode tests whether the recommended fixed design and operating policy can reach and retain the predicted optimum without crossing protective boundaries. The dynamic state may use the thermodynamic, stored-mass and rotor-speed states required by the physical model, augmented by actuator, filter, controller and energy-accumulator states.[51]

(59) $$M(x)\dot{x} = f\left(t, x, u, z, p\right), 0 = h\left(t, x, u, z, p\right)$$

(60) $$\frac{\mathrm{d}M_k}{\mathrm{d}t} = \dot{m}_{\mathrm{in,k}} - \dot{m}_{\mathrm{out,k}} + \dot{m}_{\mathrm{source,k}} - \dot{m}_{\mathrm{sink,k}}$$

(61) $$\frac{\mathrm{d}M_{\mathrm{v},2}}{\mathrm{d}t} = \dot{m}_{\mathrm{v,in}} + \dot{m}_{\mathrm{evap}} - \dot{m}_{\mathrm{v,out}} - \dot{m}_{\mathrm{cond}}$$

(62) $$\frac{\mathrm{d}U_{2,\mathrm{stored}}}{\mathrm{d}t} = \dot{H}_{\mathrm{in}} + \dot{Q}_{\mathrm{wall}} - \dot{H}_{\mathrm{out}} - \dot{W}_{\mathrm{sh}} - \dot{H}_{\mathrm{liquid,out}}$$

(63) $$J_j \frac{\mathrm{d}\Omega_j}{\mathrm{d}t} = \tau_{turbine,j} - \tau_{generator,j} - \tau_{loss,j}$$

[51] Astrom, K.J. and Murray, R.M. (2008) *Feedback Systems: An Introduction for Scientists and Engineers*. Princeton: Princeton University Press.

(64) $$\tau_a \frac{\mathrm{d}u_a}{\mathrm{d}t} = \mathrm{clip}(u_{cmd}, u_{min}, u_{max}) - u_a$$

(65) $$x_{meas,k} = x_{true,k} + b_k + n_k, \tau_f \dot{x}_{f,k} = x_{meas,k} - x_{f,k}$$

(66) $$x_{c,k+1} = G_c(x_{c,k}, y_{meas,k}, \mathrm{setpoints}, \Delta t_c)$$

(67) $$\mathrm{err}_{step} = \frac{e_{local}}{atol + rtol max(|x_n|, |x_{n+1}|)_2}$$

A stiff implicit integrator such as BDF or Radau is recommended because evaporation, pressure, sensors, actuators and rotor inertia may create widely separated time scales. Event functions shall stop or branch the integration at low Zone 2 pressure, overspeed, excessive carryover, temperature limit, map exit, generator trip or loss of a critical signal. The independent protective trip is modelled as an event that overrides the normal controller and optimiser.

Dynamic test	**Mandatory disturbance or transition**
Start-up	Zero-flow or bypass state to normal flow; enable pump, injection and generator loads in the commissioned sequence
Inlet step/ramp	Velocity, pressure, temperature and humidity steps plus realistic ramps
Atmospheric change	Pressure and temperature changes over the design envelope
Injection upset	Valve stiction, pump lag, nozzle blockage, excess flow and loss of flow
Electrical upset	Load rejection, converter trip, grid loss, generator torque saturation
Sensor upset	Bias, drift, frozen value, noise, single-channel failure and Zone 2 disagreement
Protection	Low P_2, overspeed, high vibration/carryover proxies, emergency stop and safe-state proof
Restart	Manual reset only after causes clear and stable dwell criteria are satisfied

7. Fixed-design optimisation algorithm

The fixed-design optimiser chooses the physical parameters that maximise generated net energy over the specified scenario set while preserving all hard constraints. The optimiser is nested: for every candidate design, an inner optimiser first selects the best feasible operating variables for each scenario.

(68) $$d \in D = [d_{\min}, d_{\max}] \times D_{\mathrm{discrete}}$$

(69) $$J_E(d) = \sum_s w_s E_{\mathrm{net,s}}(d, u_s(d))$$

(70) $$\Pi_g(d) = \sum_s \sum_j \lambda_j \max[0, g_j(d, u_s, z_s)]^2$$

where $\Pi_g(d)$ is a penalty function.

(71) $$J_{\mathrm{robust}}(d) = \alpha E_{\mathrm{expected}}(d) + (1-\alpha) E_{\mathrm{worst}}(d) - \Pi_g(d) - \lambda_C C_{\mathrm{norm}}(d)$$

(72) $$J_{\mathrm{robust}}(d) \text{subject to} g_j \le 0 \text{ for every hard constraint}$$

(73) $$\sum_s w_s = 1, w_s \ge 0$$

The default primary objective is energy. C_{norm} is an optional normalised complexity, capital or footprint term used only when the user activates a secondary design objective. Hard safety and physical constraints are never traded against energy by a finite economic penalty; any hard violation makes the candidate ineligible.

7.1 Recommended hybrid search

1. Enumerate small discrete sets such as turbine family, nozzle type, blade-count option, generator voltage class or pump family. Do not represent such choices as continuous numbers.
2. Generate an initial space-filling sample over continuous bounds using Latin-hypercube or Sobol points. Evaluate all samples at the low-cost fidelity level and preserve every result in the candidate log.
3. Run a bounded global population search, such as differential evolution, on the best regions. Parallelise candidates, not residual iterations within one candidate.[52]
4. Re-evaluate the leading feasible candidates at the engineering fidelity level. Remove candidates that relied on a low-fidelity artefact.
5. Refine each finalist with a derivative-based constrained local method such as trust-constr, SQP or interior-point optimisation. Use analytic or automatic derivatives where available and scaled finite differences otherwise.[53]
6. Perform robustness, dynamic and uncertainty tests. A candidate that has higher nominal energy but fails robustness shall not be recommended.[54]
7. Apply deterministic tie-breaking: highest robust energy, then highest minimum pressure reserve, then lowest carryover, then lower auxiliary power, then smaller/simpler hardware.

(74) $$\mathrm{rank}(d_i) = \mathrm{lexicographic}\left[-I_{\mathrm{hard}}, E_{\mathrm{worst}}, E_{\mathrm{expected}}, m_{\mathrm{P,min}}, -f_{\mathrm{carry,max}}, -\dot{W}_{\mathrm{aux}}, -C_{\mathrm{norm}}\right]$$

(75) $$\mathrm{KKT}_{\mathrm{res}} = \left\|\nabla_d L(d,\lambda,\nu)\right\|_\infty + \left\|\max\left(g(d),0\right)\right\|_\infty + \left\|h(d)\right\|_\infty$$

(76) $$\mathrm{stop}_{\mathrm{design}} \Leftrightarrow \mathrm{KKT}_{\mathrm{res}} \le \varepsilon_{\mathrm{KKT}} \vee \frac{\left|J_k - J_{k\text{-}1}\right|}{1+\left|J_{k\text{-}1}\right|} \le \varepsilon_J \text{ for } N_{\mathrm{stall}} \text{ iterations}$$

7.2 Design variables and bounds

Variable	Type	Optimisation role
A_1 or D_1	Continuous	Scale/flow capture; prescribed in strict forward mode or optimised in design mode
$r_A = A_2/A_1$	Continuous	Controls Zone 2 velocity, residence time, diameter and pressure loss
L_{Z2} or V_{Z2}	Continuous	Must satisfy mixing and evaporation residence-time requirements
$D_{\mathrm{tip,j}}$ and $\lambda_{\mathrm{h,j}}$	Continuous plus component family	Determine annulus area, speed, tip speed and structural scale
Turbine map/family	Discrete	Must contain the operating envelope for both stages
$D_{32,\mathrm{target}}$	Continuous within atomiser capability	Controls evaporation time, carryover and pump/nozzle requirement
n_{or} and $A_{\mathrm{or,total}}$	Discrete + continuous	Nozzle count and total flow area
D_l and line routing parameters	Continuous/discrete	Pressure loss, response time and liquid velocity
$V_{\mathrm{reservoir}}$	Continuous or derived	Unattended duration and reserve factor
Pump family/rating	Discrete	Flow, pressure, efficiency, NPSH and minimum-flow envelope
Generator/drive family and rating	Discrete	Efficiency, torque-speed envelope, current and overspeed/load rejection behaviour
Wall/shaft dimensions	Continuous or derived	Preliminary structural constraints; final design requires dedicated analysis

[52] Storn, R. and Price, K. (1997) Differential evolution - a simple and efficient heuristic for global optimization over continuous spaces, *Journal of Global Optimization*, 11, pp. 341-359.
[53] Nocedal, J. and Wright, S.J. (2006) *Numerical Optimization*. 2nd edn. New York: Springer.
[54] Saltelli, A. et al. (2008) *Global Sensitivity Analysis*: The Primer. Chichester: Wiley.

8. Optimal operating-variable algorithm

For each fixed design and scenario, the inner optimiser determines the operational variables that maximise net power or integrated net energy. Its domain is strictly limited by the design ratings, turbine maps, pressure factor interval, liquid capacity, actuator limits and protective margins.

(77) $$u_{\mathrm{op}} = \left[F_P, \dot{m}_l, N_{1,\mathrm{set}}, N_{2,\mathrm{set}}, \tau_{\mathrm{g},1}, \tau_{\mathrm{g},2}, x_{\mathrm{pitch},1}, x_{\mathrm{pitch},2}\right]^T$$

(78a) $$\dot{W}_{\mathrm{net}}(d, u, z_s)$$

(78b) $$\int_0^{t_{\mathrm{op}}} \dot{W}_{\mathrm{net}}[x(t), u(t), z(t)]\mathrm{d}t$$

(79) $$U(d,z) = \{u : F_{\mathrm{P,min}} \le F_P \le F_{\mathrm{P,max}}, \dot{m}_{l,min} \le \dot{m}_l \le \dot{m}_{l,max}, map_j \in T_j, g_{hard} \le 0\}$$

(80) $$J_{\mathrm{MPC}} = -\sum_{\mathrm{k}=0}^{N_p-1} \dot{W}_{\mathrm{net},k}\Delta t + \sum_{\mathrm{k}=0}^{N_p-1} \|\Delta u_k\|_R^2 + \sum_{\mathrm{k}=0}^{N_p-1} \|\max(g_k, 0)\|_Q^2$$

(81) $$u_{\mathrm{k}+1} = \Pi_U\left[u_k + \alpha_k \nabla_u \dot{W}_{\mathrm{net}}\right]$$

where Π_U is a projection operator that takes the proposed gradient step and project it back into the feasible operating set U.

(82) $$F_{\mathrm{P,schedule}}(z) = \mathrm{clip}\left[F_{\mathrm{P,base}} + \mathrm{K}_U(U_1 - U_{\mathrm{ref}}) + \mathrm{K}_T(T_1 - T_{\mathrm{ref}}) + K_\phi(\phi_1 - \phi_{\mathrm{ref}}), F_{\mathrm{P,min}}, F_{\mathrm{P,max}}\right]$$

(83) $$s_j \ge s_{\mathrm{j,min}}$$

A bounded coordinate or pattern search is recommended when the physical model is noisy, table-based or non-smooth. A gradient-based method is appropriate when the residual solution and maps are differentiable. In dynamic closed-loop tests, the operating optimiser is slower than the pressure controller and generator governor and is suspended whenever stability, measurement quality or constraint reserve is inadequate.[55]

Operating variable	Typical effect	Important coupled constraints
F_P	Allocates pressure drop between stages and sets the Zone 2 pressure target	P_1 head, second-stage pressure drop, atmospheric margin, mixing losses
$\dot{m}_l$ / valve-pump command	Controls cooling, humidity and first-stage downstream state	Saturation, evaporation capacity, carryover, pump power, P_2 reserve
N_1 / generator 1 torque	Moves turbine 1 operating point and electrical extraction	Map, overspeed, torque, pressure and flow interaction
N_2 / generator 2 torque	Moves turbine 2 operating point and outlet resistance	Map, backflow, overspeed and outlet pressure
Pitch/stator settings	Changes flow coefficient and map location	Actuator rate, stall/choke, structural and control limits
Drain/bypass	Controls liquid inventory or protects pressure/flow	Energy loss, emissions, safe state and equipment-specific rules

[55] Rawlings, J.B., Mayne, D.Q. and Diehl, M. (2017) *Model Predictive Control: Theory, Computation, and Design*. 2nd edn. Madison: Nob Hill Publishing.

9. Determination of the recommended system parameters

The simulator shall not identify the recommendation from nominal power alone. It shall compare feasible finalist designs using energy, constraint reserve, robustness and confidence in the underlying component data. The selected recommendation includes both the fixed design d and an operational schedule $u(z)$.

(84) $$S_{\mathrm{E,i}} = \frac{E_i - E_{\mathrm{min}}}{E_{\mathrm{max}} - E_{\mathrm{min}} + \varepsilon}$$

(85) $$m_i = \min_{s,j} \frac{g_j(d_i, u_s, z_s)}{s_{scale,j}}$$

(86) $$P_{\mathrm{feas,i}} = \frac{1}{N_r} \sum_r 1[g_{\mathrm{max}}(d_i, u_i, \xi_r) \le 0]$$

(87) $$\{E_{\mathrm{expected,i}} : P_{\mathrm{feas,i}} \ge P_{\mathrm{req}}, E_{\mathrm{worst,i}} > 0, m_i \ge m_{\mathrm{req}}\}$$

(88) $$C_{\mathrm{conf}} = C_{\mathrm{data}} C_{\mathrm{solver}} C_{\mathrm{sensitivity}} C_{\mathrm{validation}}$$

(89) $$e_{\mathrm{y,x}} = \frac{x}{y} \frac{\partial y}{\partial x}$$

(90) $$d_a \succ_P d_b \Leftrightarrow (E_a \ge E_b) \wedge (m_a \ge m_b) \wedge (\dot{W}_{\mathrm{aux,a}} \le \dot{W}_{\mathrm{aux,b}}), \text{with at least one strict inequality}$$

The recommendation report shall identify any parameter that is sitting on a bound. A bound-active recommendation is a signal that either the search domain is too narrow or a physical constraint controls the optimum. The simulator shall distinguish these cases.

Recommendation item	Required content
Fixed geometry	All principal dimensions, tolerances assumed, material/structural placeholders and active geometric bounds
Turbines and generators	Selected map/family, design point, speed/torque/power ratings, map margin and generator/drive rating
Spray and liquid system	Liquid, nominal and maximum flow, droplet target, nozzle count/area, pump duty, pipe size and reservoir
Nominal setpoints	F_P , $P_{2,\mathrm{set}}$, $\dot{m}_l$, N_1/N_2 or load/torque setpoints and expected net power
Operating schedule	Optimal setpoints over each scenario or fitted schedule versus measured conditions
Performance envelope	Expected, minimum and maximum power/energy and all limiting constraints
Confidence and assumptions	Measured, manufacturer, literature or assumed data; sensitivity and uncertainty impact
Required validation	CFD points, turbine-map tests, spray tests, structural analyses and dynamic/protection tests still outstanding

10. Performance testing of the recommended system

After selecting d_{rec} , the simulator shall perform a fresh verification campaign without reusing the coarse optimisation approximations. The final test uses the highest available fidelity, tight solver tolerances and a locked reproducibility seed.

(91) $$Z_{\mathrm{test}} = Z_{\mathrm{designpoints}} \cup Z_{\mathrm{offdesigngrid}} \cup Z_{\mathrm{extremes}} \cup Z_{\mathrm{transients}} \cup Z_{\mathrm{uncertainty}}$$

(92) $$P_{\text{feas}} = \frac{1}{N}\sum_i 1\left[g_{\max,i} \le 0\right]$$

(93) $$E_{expected} = \sum_s w_s E_s \qquad E_{worst} = \min_s E_s$$

(94) $$\text{CVaR}_\alpha(\text{loss}) = \text{E}\left[\text{loss} \vee \text{loss} \ge \text{VaR}_\alpha(\text{loss})\right]$$

(95) $$\sigma_E^2 = \frac{1}{N-1}\sum_i (E_i - E)^2$$

(96) $$\varepsilon_{\text{energy}} = \frac{\left|\dot{H}_{\text{in}} + \dot{Q}_{\text{in}} - \dot{H}_{\text{out}} - \dot{W}_{\text{sh}} - \text{d}U_{\text{stored}}/\text{dt}\right|}{\max\left(\left|\dot{H}_{\text{in}}\right|, \varepsilon\right)}$$

(97) $$\eta_{\text{net}} = \frac{\dot{W}_{\text{net}}}{\dot{W}_{\text{available,pressure}} + \dot{W}_{\text{available,kinetic}} + \dot{W}_{\text{available,thermal}}}$$

(98) $$\text{CF} = \frac{E_{\text{net}}}{\dot{W}_{\text{net,rated}} t_{\text{op}}}$$

(99) $$R_{\text{design}} = \min\left(m_{\text{pressure}}, m_{\text{map}}, m_{\text{tip}}, m_{\text{struct}}, m_{\text{carry}}, m_{\text{electrical}}, m_{\text{NPSH}}\right)$$

(100) $$\text{CI}_{95}(P_{\text{feas}}) \approx P_{\text{feas}} \pm 1.96\sqrt{\frac{P_{\text{feas}}(1-P_{\text{feas}})}{N}}$$

Test family	**Minimum test set**	**Pass criterion**
Nominal design point	Expected inlet and ambient conditions	Converged; positive net power; all target reserves met
Pressure-factor sweep	Full feasible F_P interval	Clear optimum or plateau; no hidden discontinuity; P_2/P_3 constraints maintained
Injection sweep	Zero to maximum permitted liquid flow	Energy optimum identified; saturation and carryover boundaries resolved
Environmental grid	Min/nominal/max $U_1, P_1, T_1, \phi_1, P_a, T_a, \phi_a$	No unreported extrapolation; operating schedule feasible where claimed
Turbine map sweep	Corrected flow and speed neighbourhoods	No stall/choke/map-domain violation at recommended points
Uncertainty/Monte Carlo	Losses, efficiencies, K_{evap}, sensor bias, auxiliary demand	Required P_{feas} and worst-case power met
Sensitivity	Local derivatives and global variance screening	Dominant assumptions identified and reported
Dynamic transients	Start-up, ramps, steps, load rejection, sensor and actuator faults	Protection works; no unsafe optimiser action; stable recovery or safe trip
Conservation	Mass, species, momentum/pressure and energy residuals	Residuals below configured tolerances
Repeatability	Same input and seed on same solver version	Identical recommendation within numerical tolerance

11. Text logs, saved results and figures

Every run shall create a unique output directory and shall write incrementally so that information survives interruption. Text logs are mandatory. Machine-readable JSON/CSV/HDF5 files may be written in addition, but not instead of the text logs requested here.

Data, solver and output separation

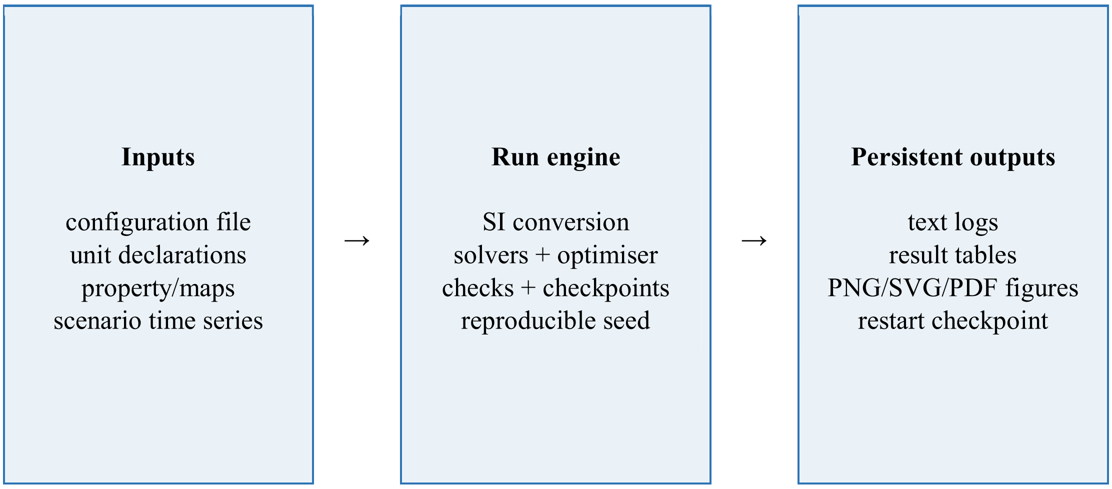

Figure 11.1. Separation of inputs, run engine and persistent outputs.

11.1 Directory and file convention

(101) $$\text{run\_name} = \text{YYYYMMDD\_HHMMSS\_project\_mode\_seed}$$

(102) $$\text{file\_name} = \text{concat}\left(\text{run\_name}_{,}\ \text{scenario}_{,}\ \text{content}_{,,}\ \text{extension}\right)$$

Required output-directory pattern

```
run_20260701_143025_prototypeA_designopt_seed42/
  run_manifest.txt
  input_echo.txt
  validation_log.txt
  solver_log.txt
  optimisation_log.txt
  constraint_log.txt
  scenario_results.txt
  recommended_design.txt
  warnings_and_failures.txt
  checkpoint.json
  results.csv
  figures/
    pressure_temperature_profile.png
    power_vs_pressure_factor.png
    power_vs_injection.png
    optimisation_convergence.png
    constraint_reserves.png
    turbine_map_stage1.png
    turbine_map_stage2.png
    evaporation_regime_residence.png
    sensitivity_tornado.png
    uncertainty_distribution.png
    transient_zone2_pressure.png
    energy_balance.png
```

Text file	Minimum content and write policy
run_manifest.txt	Software and schema versions, platform, date/time, project, mode, random seed, solver/optimiser settings, selected liquid-property formulation and

Text file	Minimum content and write policy
	version, declared property domain, flash and high-temperature model settings, and git/build hash where available; written first
input_echo.txt	Original values and units, converted SI values, assumed/default parameters and rejected fields
validation_log.txt	All range, dimensional, pressure-reference, critical-pressure, liquid-temperature, property-domain, map-domain and cross-field checks
solver_log.txt	Continuation step, nonlinear and coupled energy–mass iteration, relaxation factor, residual norm, step norm, Jacobian/line-search status, active evaporation regime, active limiting constraint and final state
optimisation_log.txt	Candidate, generation/iteration, design variables, inner optimum, objective, feasibility, penalties, accepted/rejected status and elapsed time
constraint_log.txt	Every constraint value, limit, reserve and active/inactive flag, including property-domain, minimum temperature, evaporation, finite-capacity, carryover and regime-transition checks, for each final scenario and failed finalist
scenario_results.txt	One human-readable block per scenario containing selected operating variables, states, power, energy, status, $T_{sat,2}$, $h_{fg,2}$, gas and liquid superheat, flash fraction and flow, gas-driven evaporation, K_{eff} , representative droplet temperature, active regime and active limiting constraint
recommended_design.txt	Final fixed design, operating schedule, ratings, performance envelope, confidence, assumptions and validation actions
warnings_and_failures.txt	Property errors, critical-pressure rejection, property-domain violations, failed coupled iterations, solver failures, extrapolation attempts, nonphysical states, bound-active parameters and protection events; append immediately
checkpoint.json	Restartable optimiser population, best candidates, random-state information and counters; atomic replace after each checkpoint interval

11.2 Required figures

Figure	Required content
Station pressure and temperature profile	P, T versus station/axial location; ambient and set/trip references
Power versus pressure factor	Gross stage powers, pump/auxiliary demand and net power across F_P
Power versus injection	Net power, T_2, P_2 reserve, saturation and carryover versus $\dot{m}_l$
Optimisation convergence	Best/median objective, feasible count, maximum violation and evaluation count
Constraint reserves	Minimum reserve by constraint and scenario; zero line clearly marked
Turbine maps	Corrected operating points, trajectories and map boundaries for each turbine
Evaporation regime and residence	D_{32} , t_{evap} , t_{res} , total evaporation fraction and carryover over the operating envelope; T_{1e} , T_2 and $T_{sat,2}$; gas and liquid superheat; flash and gas-driven evaporation; K_{eff} ; active evaporation regime; active limiting constraint; and the applicable property-domain boundary
Energy balance	Stage outputs, spray heat transfer, pump/auxiliary losses and net output
Sensitivity tornado	Local or global effect of dominant inputs on energy and recommended

Figure	Required content
	dimensions
Uncertainty distribution	Histogram/CDF of net energy and feasibility; percentiles and worst cases
Transient response	$P_2, P_{2,\mathrm{set}}, P_{2,\mathrm{trip}}, T_2, \dot{m}_l$, speeds, loads, net power and trip state versus time
Pareto front	Energy versus robustness/auxiliary/size when secondary objectives are enabled

Figures shall be saved at a minimum as PNG. Vector SVG or PDF shall also be saved for line plots when supported. Plot data shall be saved in a machine-readable table so every figure can be regenerated. File names and axis labels shall include units from the selected display system, while the underlying data table retains SI values and unit metadata.

$$\mathrm{figure\_data} = \{x_{\mathrm{SI}}, \mathrm{y}_{\mathrm{SI}}, \mathrm{unit}_x, \mathrm{unit}_y, \mathrm{x}_{\mathrm{display}}, \mathrm{y}_{\mathrm{display}}, \mathrm{scenario\_id}, \mathrm{run\_id}\} \qquad (103)$$

12. Numerical solution, scaling and failure recovery

12.1 Variable and residual scaling

(104) $$y_i = \frac{y_i - y_{\mathrm{ref,i}}}{S_{\mathrm{y,i}}}$$

(105) $$F_i = \frac{F_i}{S_{\mathrm{F,i}}}$$

Scale factors shall be physically meaningful, such as $10^5\,\mathrm{Pa}$ for pressure, $300\mathrm{K}$ for temperature, nominal mass flow for flow and rated power for power. Poor scaling can make a mathematically solvable point appear singular.

12.2 Derivatives and merit function

(106) $$J_F[:,i] \approx \frac{F(y+h_i e_{i\,i}) - F(y-h_i e_i)}{2h_i}$$

(107) $$\Psi(y) = \frac{1}{2}\|F(y)\|_2^2 + \mu\sum_j \max[0, g_j(y)]^2$$

(108) $$y_{\mathrm{k+1}} = y_k + \alpha_k p_k, \Psi(y_{\mathrm{k+1}}) \le \Psi(y_k) + c_1\alpha_k \nabla\,\Psi^T p_k$$

(109) $$\Delta_{\mathrm{k+1}} = \gamma_{\mathrm{inc}}\Delta_k \mathrm{if} \rho_k > \rho_{\mathrm{hi}}, \Delta_{\mathrm{k+1}} = \gamma_{\mathrm{dec}}\Delta_k \text{ if } \rho_k < \rho_{\mathrm{lo}}$$

(110) $$\mathrm{root\_stop} \Leftrightarrow \|F\|_\infty \le \varepsilon_F \wedge \|\Delta y\|_\infty \le \varepsilon_y$$

(111) $$\mathrm{opt\_stop} \Leftrightarrow \frac{|\Delta \mathrm{J}|}{1+|J|} \le \varepsilon_J \wedge \|\mathrm{projectedgradient}\|_\infty \le \varepsilon_{\mathrm{grad}} \wedge g_{\max} \le \varepsilon_g$$

12.3 Failure-recovery sequence

1. Retry from the last converged continuation point with a smaller continuation step.
2. Rescale residuals and variables if the Jacobian condition estimate exceeds the configured threshold.
3. Switch from Newton/SQP to bounded least squares or trust-region reflective solving.
4. Reduce the operating-variable step toward a known feasible point.
5. Use the sequential fallback to regenerate an initial guess, then return to the simultaneous solve.
6. If a property call is outside its valid domain, do not extrapolate silently; mark the candidate invalid and log the requested state.
7. If all retries fail, return NONCONVERGED and a diagnostic package. Do not assign an arbitrary large negative power while claiming physical feasibility.

13. Complete simulator pseudocode

The following pseudocode is deliberately implementation-oriented. It defines the order of operations, persistence and nested optimisation required to satisfy the simulator specification.

Algorithm 13.1 - main program

```
PROCEDURE MAIN(config_path):
    config_raw  <- load_configuration(config_path)
    schema      <- validate_schema_version(config_raw)
    run_id      <- create_run_id(config_raw.project, config_raw.mode, config_raw.seed)
    paths       <- create_output_directory(run_id)
    open_incremental_text_logs(paths)

    write_run_manifest(config_raw, schema, software_versions, platform)

    TRY:
        config_si <- convert_all_inputs_to_SI(config_raw, strict_units = TRUE)
        validate_cross_field_rules(config_si)
        write_input_echo(config_raw, config_si)

        property_model <- initialise_property_model(config_si)
        component_maps <- load_and_validate_maps(config_si)
        scenarios      <- build_scenarios(config_si)
        uncertainty    <- build_uncertainty_sampler(config_si)
        solver_options <- build_solver_options(config_si)

        IF mode == FORWARD:
            result <- simulate_fixed_design(config_si.design, config_si.operation,
                                            scenarios, solver_options)

        ELSE IF mode == INVERSE:
            result <- solve_inverse_sizing(config_si.target, config_si.bounds,
                                           scenarios, solver_options)

        ELSE IF mode == OPERATING_OPT:
            result <- optimise_operation_for_design(config_si.design, scenarios,
                                                    solver_options)

        ELSE IF mode == DESIGN_OPT:
            result <- optimise_fixed_design(config_si, scenarios, uncertainty,
                                            solver_options)

        ELSE IF mode == DYNAMIC_TEST:
            result <- run_dynamic_test_suite(config_si.design,
                                             config_si.controller,
                                             scenarios, solver_options)

        write_final_text_reports(result)
        save_machine_readable_results(result)
        generate_all_required_figures(result)
        validate_output_files(paths)
        close_logs(status = SUCCESS)
        RETURN result

    EXCEPT InputOrUnitError AS e:
        append_warning_and_failure(e, stage = PREPROCESS)
        close_logs(status = INPUT_FAILURE)
        RAISE

    EXCEPT Exception AS e:
        save_emergency_checkpoint()
        append_warning_and_failure(e, stage = UNKNOWN)
        close_logs(status = ABORTED)
        RAISE
```

Algorithm 13.2 - steady physical kernel

```
FUNCTION SOLVE_STEADY(design d, operation u, scenario z, parameters p):
    z <- psychrometric_preprocess(z, p)
    bounds <- compute_feasible_pressure_and_operating_bounds(d, z, p)
    IF bounds.empty:
        RETURN status(INFEASIBLE_PRECHECK, reason = bounds.reason)

    u <- project_to_bounds(u, bounds)
    y0 <- sequential_initial_guess(d, u, z, p)

    FOR continuation_step k = 0 TO N_continuation:
```

```
        lambda <- k / N_continuation
        target <- continuation_target(u, lambda)
        solve <- bounded_nonlinear_residual_solve(F, y0, d, target, z, p)
        log_solver_iteration_history(solve)

        IF NOT solve.converged:
            solve <- recovery_sequence(solve, y0, d, target, z, p)
        IF NOT solve.converged:
            RETURN status(NONCONVERGED, diagnostics = solve.diagnostics)
        y0 <- solve.y

    outputs     <- calculate_derived_outputs(solve.y, d, u, z, p)
    constraints <- evaluate_all_constraints(outputs, d, u, z, p)
    conservation<- evaluate_conservation_residuals(outputs)

    IF any_hard_violation(constraints):
        RETURN status(CONVERGED_INFEASIBLE, outputs, constraints, conservation)
    ELSE:
        RETURN status(CONVERGED_FEASIBLE, outputs, constraints, conservation)
```

Algorithm 13.3 - inner operating optimisation

```
FUNCTION OPTIMISE_OPERATION(design d, scenario z):
    U <- construct_operating_domain(d, z)
    feasible_seeds <- generate_operating_seeds(U)
    best <- NONE

    FOR seed IN feasible_seeds:
        candidate <- bounded_pattern_or_gradient_search(
                         objective = -net_power,
                         variables = [F_P, liquid_flow, speed_or_load_1,
                                      speed_or_load_2, pitch_variables],
                         bounds = U,
                         evaluator = SOLVE_STEADY)
        write_optimisation_log(candidate)
        IF candidate.feasible AND better(candidate, best):
            best <- candidate

    IF best IS NONE:
        RETURN status(NO_FEASIBLE_OPERATION)
    RETURN best
```

Algorithm 13.4 - fixed-design optimisation

```
FUNCTION OPTIMISE_FIXED_DESIGN(config, scenarios, uncertainty):
    discrete_sets <- enumerate_discrete_component_choices(config)
    archive <- empty_candidate_archive()

    FOR components IN discrete_sets:
        population <- space_filling_initial_population(config.design_bounds,
                                                       components,
                                                       seed = config.seed)
        population <- global_population_search(
            population,
            evaluator = EVALUATE_DESIGN_LOW_FIDELITY,
            checkpoint_callback = save_checkpoint,
            log_callback = write_optimisation_log)
        archive.add(population)

    finalists <- select_feasible_diverse_finalists(archive)

    FOR d IN finalists:
        d_refined <- constrained_local_refinement(
                         d,
                         evaluator = EVALUATE_DESIGN_ENGINEERING_FIDELITY)
        robust <- RUN_ROBUSTNESS_CAMPAIGN(d_refined, scenarios, uncertainty)
        dynamic <- RUN_DYNAMIC_TEST_SUITE(d_refined)
        archive.update(d_refined, robust, dynamic)

    eligible <- filter(archive,
                       hard_violations = 0,
                       feasibility_probability >= P_required,
                       worst_case_net_energy > 0,
                       minimum_reserve >= reserve_required)

    IF eligible.empty:
        RETURN status(NO_RECOMMENDATION, best_near_feasible = archive.best_diagnostic())

    recommended <- lexicographic_select(eligible)
```

```
    operating_schedule <- fit_or_tabulate_optimal_schedule(recommended)
    RETURN package_recommendation(recommended, operating_schedule, archive)
```

Algorithm 13.5 - robustness campaign

```
FUNCTION RUN_ROBUSTNESS_CAMPAIGN(design d, scenarios, uncertainty):
    results <- []
    FOR deterministic scenario z IN scenarios:
        u_star <- OPTIMISE_OPERATION(d, z)
        results.append(SOLVE_STEADY(d, u_star, z))

    samples <- draw_reproducible_samples(uncertainty)
    FOR sample xi IN samples:
        z_xi <- apply_uncertainty(scenarios, xi)
        u_xi <- schedule_or_reoptimise_operation(d, z_xi)
        results.append(SOLVE_STEADY(d, u_xi, z_xi))

    calculate_expected_worst_percentile_metrics(results)
    calculate_constraint_probabilities(results)
    calculate_local_and_global_sensitivities(results)
    save_scenario_results_incrementally(results)
    RETURN results
```

14. Software architecture and implementation requirements

Module	Required responsibilities
units	Unit registry, dimensional validation, absolute/gauge/differential pressure handling, original/SI/display conversion
schema	Input-file versioning, defaults, required fields, bounds, enumerations and migration
properties	Humid-air and liquid properties, saturation pressure, validity range and uncertainty
components	Turbine maps, pump curves, generator/drive maps, valve/nozzle correlations and structural placeholders
model_steady	The implemented thermodynamic, psychrometric, pressure-loss, turbine, spray, shaft and electrical equations, together with equations (40)-(58) of this Annexure, residuals, constraints and derived outputs
model_dynamic	DAE/ODE states, controller, sensors, actuators, trips and event handling
solvers	Scaling, continuation, root/least-squares methods, derivatives, recovery and diagnostics
optimise_operation	Bounded inner optimisation, operating schedule and map-aware projections
optimise_design	Discrete enumeration, global population search, local refinement, archive and checkpoints
robustness	Scenario grids, uncertainty sampling, sensitivity, percentiles and dynamic test suite
logging	Incremental text logs, structured result data, warnings, atomic checkpoints and reproducibility metadata
plots	Required figures, data export, SI/display labels and deterministic file naming
report	Human-readable recommendation, assumptions, active constraints, confidence and validation requirements
tests	Unit, property, conservation, regression, optimisation and failure-mode tests

The simulator should provide both a command-line interface and an optional graphical interface. The command-line interface is the authoritative reproducible path. The graphical interface must write an equivalent configuration file rather than passing hidden state directly to the solver.

Illustrative command-line interface

```
simulator run project.yaml --mode design-opt --units SI --seed 42
```

```
simulator run project_us.yaml --mode forward --units US_CUSTOMARY
simulator validate project.yaml
simulator resume run_20260701_143025_prototypeA_designopt_seed42/checkpoint.json
simulator plot run_directory --units US_CUSTOMARY
```

14.1 Example dual-unit input records

The following fragments illustrate format only. The values are not a validated design point and must not be interpreted as recommended operating conditions.

Example SI input

```
unit_system: SI
mode: DESIGN_OPT
boundary:
  inlet_velocity:       {value: 35.0, unit: "m/s"}
  inlet_pressure:       {value: 125.0, unit: "kPa(a)"}
  inlet_temperature:    {value: 60.0, unit: "degC"}
  inlet_relative_humidity: {value: 20.0, unit: "%"}
  ambient_pressure:     {value: 101.325, unit: "kPa(a)"}
  ambient_temperature:  {value: 25.0, unit: "degC"}
  ambient_relative_humidity: {value: 50.0, unit: "%"}
operation:
  duration:              {value: 8.0, unit: "h"}
optimisation:
  pressure_factor_bounds: [1.01, 1.18]
  seed: 42
```

Equivalent US customary input

```
unit_system: US_CUSTOMARY
mode: DESIGN_OPT
boundary:
  inlet_velocity:       {value: 78.293, unit: "mph"}
  inlet_pressure:       {value: 18.130, unit: "psia"}
  inlet_temperature:    {value: 140.0, unit: "degF"}
  inlet_relative_humidity: {value: 20.0, unit: "%"}
  ambient_pressure:     {value: 14.696, unit: "psia"}
  ambient_temperature:  {value: 77.0, unit: "degF"}
  ambient_relative_humidity: {value: 50.0, unit: "%"}
operation:
  duration:              {value: 8.0, unit: "h"}
optimisation:
  pressure_factor_bounds: [1.01, 1.18]
  seed: 42
```

15. Verification, calibration and acceptance of the simulator

Verification establishes that the equations and algorithms are implemented correctly. Validation establishes whether the model predicts the physical device. The simulator shall report these as separate statuses.

(112a) $$\varepsilon_{\text{mass}} = \frac{\left|\dot{m}_{\text{in}} + \dot{m}_{\text{added}} - \dot{m}_{\text{out}} - \dot{m}_{\text{drain}} - \dot{m}_{\text{carry}}\right|}{\max\left(\dot{m}_{\text{in}}, \varepsilon\right)}$$

(112b) $$\varepsilon_{\text{species}} = \frac{\left|\dot{m}_{\text{v,in}} + \dot{m}_{\text{evap}} - \dot{m}_{\text{v,out}} - \dot{m}_{\text{cond}}\right|}{\max\left(\dot{m}_{\text{v,out}}, \varepsilon\right)}$$

(113) $$\varepsilon_{\text{regression}} = \frac{\left|y_{\text{new}} - y_{\text{reference}}\right|}{\max\left(\left|y_{\text{reference}}\right|, y_{\text{floor}}\right)}$$

(114) $$\text{RMSE} = \sqrt{\frac{1}{N}\sum_i \left(y_{\text{pred,i}} - y_{\text{meas,i}}\right)^2}$$

(115) $$\sum_i w_i \left\| y_{\text{model}}\left(z_i, p\right) - y_{\text{measured},i} \right\|_2^2 + \lambda \left\| p - p_{\text{prior}} \right\|_2^2$$

Verification/validation activity	Required evidence
Unit conversion tests	Known conversions, round trips, pressure-reference tests, temperature-difference tests and rejection of incompatible dimensions
Property tests	Comparison with trusted humid-air and liquid reference points over the complete declared domain, including saturation temperature, enthalpy, latent heat, critical-pressure rejection and boundary points immediately inside and outside the valid domain
Equation unit tests	Each equation tested independently for known limiting cases and dimensions
Conservation tests	Mass, total water species and energy residuals below tolerance for every feasible result, with separate accounting for inlet vapour, accepted flash vapour, gas-driven vapour, flash mass returned to the liquid inventory, unevaporated liquid, carryover and drain
Limiting-case tests	Zero spray; zero second-stage pressure drop; equal ambient/internal pressure; dry air; conventional sub-boiling evaporation; gas temperature immediately below and above $T_{sat,2}$; liquid temperature immediately below and above $T_{sat,2}$; flash fraction approaching zero continuously at the boundary; high-temperature gas without flash; flash-assisted evaporation; finite-capacity re-entry after cooling; return of excess flash vapour to the liquid inventory; pressure at, below and above the critical-pressure limit; liquid temperature outside the declared property domain; failed coupled iteration and recovery; saturation boundary; and negligible losses
Regression tests	Locked inputs, solver version and seed reproduce reference outputs within declared tolerances
Optimiser tests	Known analytic benchmark functions, constrained toy problems, deterministic replay and bound/constraint enforcement
Map tests	Interpolation within domain, continuity, correct corrected variables and explicit rejection outside domain
Calibration	Measured pressure, temperature, flow, spray, power and dynamic-response data with parameter estimates and confidence intervals
Prototype validation	Blind prediction points not used for calibration; residuals and uncertainty coverage reported
Software review	Version control, code review, static analysis where applicable, test coverage, release tag and immutable run manifest

Engineering limitation: The final recommendation remains provisional until the turbine maps, pressure-loss model, atomisation and evaporation model, structural response, generator/load behaviour and controller dynamics are calibrated and validated. The simulator must make uncertainty visible rather than conceal missing component data behind a single optimal number.

16. Required final recommendation report

The simulator-generated recommendation report shall be sufficiently complete that another engineer can reproduce the run and understand why the candidate was selected. The report shall use the selected display units and shall include SI values in an appendix or parallel column.

1. Run identity, software version, configuration hash, date/time, unit system, solver/optimiser versions and random seed.
2. Input conditions, scenario weights, uncertainty distributions, property methods, maps, assumed constants and calibration sources.
3. Recommended fixed design with dimensions, component families/ratings, material placeholders, and every active design bound.
4. Recommended nominal operating point and the optimal operating variables for every deterministic scenario.

5. A tabulated or fitted operating schedule as a function of inlet and ambient conditions, including its valid domain.
6. Gross stage power, parasitic demand, net power, net energy, specific work, efficiency metrics and no-spray comparison.
7. Pressure, temperature, humidity, flow, evaporation, carryover, residence, Mach, Reynolds, map, tip-speed, structural, pump and electrical margins.
8. Expected, worst-case, percentile and uncertainty results; probability of feasibility and dominant sensitivities.[56]
9. Dynamic test outcomes including start-up, disturbances, protective trips, settling and safe-state proof.
10. All solver failures, rejected candidates, extrapolation attempts, assumptions, limitations and required physical validation.
11. Paths to all text logs, machine-readable data, checkpoints and figures.

Final status	Meaning
RECOMMENDED - VALIDATED MODEL	All configured requirements passed and the relevant component models have been calibrated/validated over the claimed domain
RECOMMENDED - PRELIMINARY	A feasible optimum was found but one or more component correlations or maps remain assumed or unvalidated
NO ROBUST RECOMMENDATION	Nominal feasible candidates exist but required uncertainty, reserve or dynamic criteria are not met
NO FEASIBLE DESIGN	No candidate satisfies all hard constraints within the supplied bounds
INDETERMINATE	Input, property, map or numerical information is insufficient to determine a defensible result

17. Special operable case: non-generating outlet-isolation rotor and constant-area flow path

This section defines an optional operable case within the general simulator specification. It does not replace the general two-stage model in sections 1–16. The general model remains applicable where the downstream stage extracts useful shaft or electrical work. The present special case applies where the downstream rotor is installed principally to maintain a controlled outward-flow boundary and to discourage atmospheric-air ingress into Zone 2, and where no electrical-generation credit is assigned to that rotor.

The special case shall be selected explicitly in the project configuration and identified in every input echo, result record, log, figure and recommendation report. Parameters and equations not expressly overridden below retain the definitions, units, property methods, constraints, solver statuses and validation requirements established by the general model.

17.1 Scenario selection and inheritance from the general model

The active model case is represented by:

(116) $$S_{case} \in \{G_2, I_{out}\}$$

Here G_2 denotes the general two-stage case and I_{out} denotes the outlet-isolation special case. When $S_{case} = G_2$, sections 1–16 apply without the special overrides in this section. When $S_{case} = I_{out}$, the simulator shall inherit the general humid-gas, liquid-property, evaporation, pressure-loss, turbine-1, structural, pump, control, uncertainty, logging and numerical-solution models, but shall apply the following scenario-specific rules:

- the inlet and Zone 2 flow areas are equal;
- the downstream rotor is treated as an outlet-isolation and forward-flow-control component;
- the downstream rotor receives no shaft-work or electrical-generation credit;
- Zone 2 pressure is a solved state obtained from coupled flow compatibility and is not an independently selected pressure-factor target;
- the paired dry baseline uses identical hardware and boundary conditions with the liquid-injection rate set to zero; and

[56] JCGM (2008) Evaluation of measurement data - Guide to the expression of uncertainty in measurement. *JCGM* 100:2008.

- any power required to drive, seal or control the outlet-isolation component is included as an auxiliary demand.

The general model therefore remains the parent model, and the special case is an explicit constrained projection of it rather than a separate unrelated formulation.

17.2 Constant-area geometry and component roles

For the special case, the inlet and Zone 2 areas and their equivalent circular diameters are constrained by:

(117) $$A_2 = A_1, D_2 = D_1 = \sqrt{\frac{4A_1}{\pi}}$$

The first turbine remains the sole generating stage. Its generator and load system are modelled in accordance with the general turbine-1 equations and applicable component map. The outlet-isolation rotor may be passive, mechanically biased, variable-pitch, variable-throat or actively driven, but its principal modelled functions are to:

- provide a forward swallowing-capacity relationship between Zone 2 and the atmospheric outlet;
- impose the outlet resistance needed to obtain a physically closed interstage pressure solution;
- discourage or limit reverse atmospheric flow under abnormal or transient pressure conditions; and
- provide a controllable boundary where a commissioned outlet-isolation setting is available.

The downstream electrical-generation term is fixed at:

(118) $$\dot{W}_{e,out} = 0$$

Any pneumatic enthalpy reduction, wake loss or rotor dissipation may be reported for conservation and thermal accounting, but shall not be represented as exported electrical power.

17.3 Coupled steady-state pressure and mass-flow closure

At a trial Zone 2 pressure, the first-turbine map or validated capacity relation shall determine the gas mass flow passed by turbine 1, subject to the inlet flow available from the source. The evaporating control volume then determines the accepted flash vapour, gas-driven evaporation, final mixture composition, temperature, density, velocity, carryover and associated pressure losses.

The gas-phase flow that must pass through the outlet-isolation rotor is:

(119) $$\dot{m}_{2,req} = \dot{m}_{T1} + \dot{m}_{evap}$$

where $\dot{m}_{evap}$ includes only liquid mass accepted into the gas phase. Unevaporated liquid, drain flow and separated carryover shall be accounted for separately in the total mass and water-species balances.

The outlet-isolation rotor forward capacity is represented by a calibrated map where available, or by a declared screening correlation at the lowest permitted fidelity. Zone 2 pressure is the root of the forward-flow residual:

(120) $$R_{P_2} = \dot{m}_{out}\left(P_2, T_2, P_a, N_{out}, u_{out}\right) - \dot{m}_{2,req} = 0$$

The first-turbine outlet pressure used in the turbine-1 expansion calculation is:

(121) $$P_{T1,out} = P_2 + \Delta P_{mix} + \Delta P_{duct}$$

Equations (119)–(121) shall be solved together with the spray-zone energy, species, evaporation-kinetics, equation-of-state and momentum relationships. The solver shall not lower the dry-case Zone 2 pressure by selecting an independent pressure factor. The injected and dry cases must each satisfy the same component-flow closure with the same physical hardware.

If no admissible root exists within the pressure interval established by the turbine-1 minimum drop, atmospheric boundary, outlet pressure loss, Zone 2 reserve and outlet-isolation map, the candidate shall be returned as `NO_PRESSURE_FLOW_SOLUTION` or the equivalent structured infeasible status. The solver shall not substitute the closest pressure point and label it physically feasible unless the residual is within the declared flow-balance tolerance.

17.4 Turbine-1 work and net electrical output

The turbine-1 shaft and electrical calculations retain the general map-based or engineering-fidelity formulation. In a screening formulation, the exported turbine-1 electrical power may be written as:

(122) $$\dot{W}_{e,1} = \eta_{g,1}\eta_{m,1}\dot{m}_{T1}\left[(h_1 - h_{T1,out}) + \frac{U_1^2 - U_{T1,out}^2}{2}\right)$$

The net exported electrical power for the outlet-isolation special case is:

(123) $$\dot{W}_{net} = \dot{W}_{e,1} - \dot{W}_{pump} - \dot{W}_{aux} - \dot{W}_{out,aux}$$

where $\dot{W}_{out,aux}$ is zero only when the outlet-isolation rotor and its sealing arrangement are genuinely passive. A motor, magnetic bearing, purge, seal, braking, pitch, actuation or control demand shall be entered or map-derived and shall not be omitted merely because the rotor produces no electricity.

For an injection-enabled point and its paired dry baseline, the net electrical change is:

(124) $$G_{paired} = 100\frac{\dot{W}_{net,inj} - \dot{W}_{net,dry}}{\dot{W}_{net,dry}}$$

The paired comparison shall use the same inlet and ambient conditions, duct geometry, turbine-1 hardware, outlet-isolation hardware, map versions, losses and auxiliary assumptions. Only injection-dependent operating variables may differ unless the report explicitly identifies an additional commissioned control action.

17.5 Atmospheric-ingress screening and pressure protection

The normal steady operating requirement remains outward flow with a positive Zone 2 pressure reserve above the applicable atmospheric boundary. The outlet-isolation rotor is not to be described as a positive shut-off valve unless that function is established by component testing.

For preliminary reverse-flow screening, an effective reverse-leakage area may be used:

(125) $$\dot{m}_{rev} = C_{d,rev}A_{rev}\sqrt{2\rho_a max(P_a - P_2, 0)}$$

Equation (125) is an incompressible low-differential screening relation. It shall be replaced by a compressible reverse-flow map or measured leakage characteristic whenever the pressure ratio, gas properties or rotor behaviour make the approximation unsuitable.

The reverse-leakage fraction is:

(126) $$\phi_{rev} = \frac{\dot{m}_{rev}}{\max(\dot{m}_{2,forward}, \varepsilon_m)}$$

The minimum normal constraints include:

(127) $$P_2 \geq P_a + \Delta P_{reserve}, \phi_{rev} \leq \phi_{rev,max}$$

A separate lower protective threshold shall be retained for dynamic operation. Injection shall be reduced or stopped before the protective threshold is reached. Where a driven outlet-isolation rotor is used, its commissioned response may be coordinated with the injection controller, but the associated drive demand must be included in equation (123).

Reverse-flow prevention is not validated merely because a forward-flow steady-state root exists. The required evidence includes a component leakage curve or map, transient pressure response, start-up and shutdown tests, loss-of-drive or loss-of-load tests where applicable, and confirmation that no unsafe optimiser action can defeat the protective hierarchy.

17.6 Special-case operating variables and optimisation

The special-case operating vector may be represented by:

(128) $$u_{special} = [\dot{m}_l, N_1 \; or \; \tau_{g,1}, u_{out}, u_{bypass}, u_{drain})$$

where u_{out} is included only where the outlet-isolation component has a physically adjustable speed, pitch, throat, torque or drive setting. No fictitious generator-2 power or generator-2 torque variable shall be introduced in this special case.

For a fixed design and scenario, the inner optimisation is:

(129) $$u_{special,opt} = argmax_{u_{special}} \dot{W}_{net}$$

subject to all pressure, evaporation, carryover, saturation, property-domain, map-domain, Mach-number, speed, torque, structural, pump, electrical, leakage, controller and numerical-convergence constraints. Zone 2 pressure is excluded from the decision vector because it is solved by equation (120).

The optimiser shall always evaluate the zero-injection point and shall return zero injection where no feasible non-zero rate increases net exported electricity. A positive increase in turbine-1 output alone is not the acceptance objective; the pump, outlet-isolation auxiliary and all other parasitic demands must be included.

17.7 Additional special-case inputs and outputs

In addition to the common general-model inputs, the special case shall declare:

1. the model-case identifier;
2. outlet-isolation rotor map, family or screening correlation and its validity domain;
3. effective forward throat area or flow coefficient;
4. reverse-leakage map or effective reverse-leakage area;
5. passive, biased or actively driven operating mode;
6. adjustable outlet-isolation variables and their limits, where applicable;
7. outlet-isolation auxiliary or drive demand and efficiency;
8. normal Zone 2 pressure reserve and protective threshold;
9. maximum permitted reverse-leakage fraction; and
10. the fixed area-ratio constraint $A_2 / A_1 = 1$.

Every special-case result shall report at least:

3. turbine-1 inlet and outlet states, pressure ratio, corrected map location, shaft power and electrical output;
4. solved Zone 2 pressure and flow-balance residual;
5. outlet-isolation forward capacity, pressure drop, operating setting and map margin;
6. outlet-rotor pneumatic dissipation without electrical credit;
7. reverse-leakage estimate and leakage fraction under the declared test condition;
8. injection rate, evaporation fraction, vapour addition, carryover and active evaporation limit;
9. pump, general auxiliary and outlet-isolation auxiliary demands;
10. net electrical power and paired dry-baseline change;
11. active constraints, status, assumptions and unvalidated component data; and
12. a statement that atmospheric-air exclusion has or has not been physically validated.

17.8 Verification and acceptance tests for the special case

The general verification and validation requirements remain mandatory. The following additional tests shall be performed:

1. Accounting test. Confirm that outlet-rotor electrical credit is exactly zero and that equation (123) closes to the reported net electrical output.
2. Constant-area test. Attempted non-unity area-ratio inputs shall be rejected or deterministically overridden to unity with a logged warning.
3. Pressure-solution test. Confirm that equation (120) is satisfied within the declared flow-balance tolerance for every feasible result.
4. Paired-baseline test. Confirm that the dry and injected cases use identical fixed hardware, maps, losses and source boundaries.
5. Injection sweep. Sweep from zero to the maximum permitted liquid rate and resolve evaporation, pressure, turbine-1 power and all parasitic demands.
6. Low-pressure and low-velocity test. Identify conditions where no coupled solution exists and distinguish component-sizing infeasibility from an adverse injection result.

7. Outlet-map sensitivity. Vary forward throat, discharge coefficient, speed, pitch or map uncertainty over the declared range.
8. Reverse-leakage test. Evaluate the complete declared reverse-pressure range and report whether the leakage requirement is met.
9. Auxiliary break-even test. Determine the maximum outlet-isolation auxiliary demand that preserves the predicted injection benefit.
10. Dynamic protection test. Simulate start-up, shutdown, inlet disturbances, injection upset, loss of outlet-isolation drive where applicable, and protective response.

A predicted benefit is material only when:

(130) $$\Delta \dot{W}_{net} > \max\left(\delta \dot{W}_{measurement}, \delta \dot{W}_{model}, \Delta \dot{W}_{minimum}\right)$$

where the right-hand terms represent measurement uncertainty, model uncertainty and the project’s minimum practical improvement threshold. A numerically positive change below this combined threshold shall be reported as no material difference.